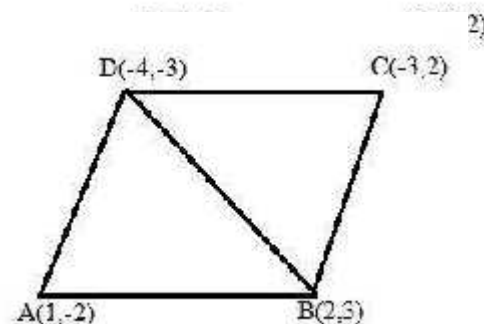


Q.1 If the points (1,-2), (2,3), (-3,2), (-4,-3) are the vertices of a parallelogram ABCD. Then taking AB as the base find height of the parallelogram.

Ans: Let the vertices of the parallelogram be A(1,-2) B(2,3) C(-3,2) and D(-4,-3).

Join BD to form two triangles ABD and BCD.



$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \triangle ABD = \frac{1}{2} [1(3 + 3) + 2(-3 + 2) + (-4)(-2 - 3)]$$

$$= \frac{1}{2} [6 - 2 + 20]$$

$$= \frac{1}{2} [24] = 12 \text{ square units.}$$

$$\text{Area of } \triangle BCD = \frac{1}{2} [2(2 + 3) + (-3)(-3 - 3) + (-4)(3 - 2)]$$

$$= \frac{1}{2} [10 + 18 - 4]$$

$$= \frac{1}{2} [24] = 12 \text{ square units.}$$

Now,

Area of parallelogram ABCD = Area of ABD + Area of BCD = 12 + 12 = 24 square units.

$$AB = \sqrt{2 - 1^2 + 3 + 2^2} = \sqrt{26}$$

$$\text{Height} = \text{Area}/\text{base} = 24/\sqrt{26}$$

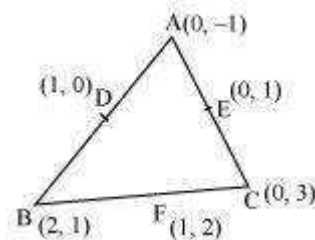
Q.2. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer: Let the vertices of the triangle be A (0, -1), B (2, 1), C (0, 3).

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2} \right) = (1, 0) \quad E = \left(\frac{0+0}{2}, \frac{3-1}{2} \right) = (0, 1) \quad F = \left(\frac{2+0}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \triangle DEF = \frac{1}{2} \{1(2 - 1) + 1(1 - 0) + 0(0 - 2)\} = \frac{1}{2} (1 + 1) = 1 \text{ square units}$$



Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$\text{Area of } \triangle ABC = \frac{1}{2} [0(1 - 3) + 2\{3 - (-1)\} + 0(-1 - 1)] = 4 \text{ square units}$$

Therefore, required ratio = 1 : 4

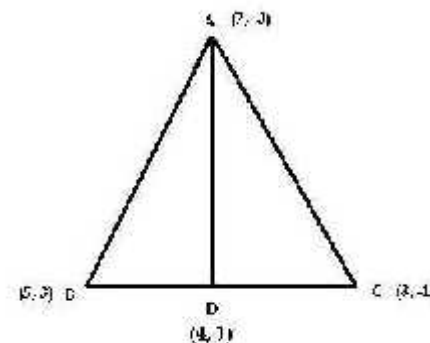
Q.3. Find the length of the median AD of the triangle ABC whose vertices are A(7,-3), B(5,3) and C(3,-1)

, where D is mid-point of the side BC

Answer: Since D is the mid point of BC, therefore,

$$D = \left(\frac{5+3}{2}, \frac{3+(-1)}{2} \right)$$

$$D = (4, 1)$$



Now length of AD can be calculated using the distance formula, i.e.,

$$AD = \sqrt{(4-7)^2 + [1-(-3)]^2} = \sqrt{(-3)^2 + (4)^2} \quad AD = 5$$

∴ The length of the median AD = 5 units

Q.4. find the ratio in which point P(-1,y) lying on the line segment joining points A(-3,10) and B(6,-8) divides it. Also find the value of y.

Solution:

Let the point $(-1, y)$ divides the line segment joining points

A $(-3, 10)$ and B $(6, -8)$ in the ratio $k:1$

Then, the coordinate of point P = $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1} \right)$

But, coordinate of point P = $(-1, y)$ (given)

$$\text{Thus, } -1 = \frac{6k-3}{k+1} \Rightarrow -k-1 = 6k-3 \Rightarrow k = \frac{2}{7}$$

$$y = \frac{-8k+10}{k+1} \Rightarrow y = \frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1} \Rightarrow y = 6$$

Hence, the ratio is 2:7 and $y = 6$

Q.5. In what ratio is the line segment joining A(2,-3) and B(5,6) divided by the x-axis ? Also , find the coordinates of the point of division.

Answer: Let the line passing through the points A(2,-3) and B(5,6) is

$$\frac{(y-6)}{(x-5)} = \frac{9}{3} \Rightarrow 3x - y - 9 = 0$$

Point where x-axis cuts this line can be obtained by putting $y = 0$

The required point is (3,0)

Now the point divides the line segment AB in the ratio

$$\frac{\sqrt{(5-3)^2 + (6-0)^2}}{\sqrt{(2-3)^2 + (-3-0)^2}} = \frac{\sqrt{4+36}}{\sqrt{1+9}} = \frac{\sqrt{40}}{\sqrt{10}} = \sqrt{4} = \frac{2}{1}$$

Q.6. line segment joining the points A (3,2) and B (5,1) is divided at the point P in ratio 1:2 and it lies on the line $3x-18y+k=0$

Answer: P divides the line segment joining the points A(3,2) and B(5,1) in the ratio 1:2

The coordinates of P can be found put by the section formula

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Here $m : n = 1:2$

$$(x_1, y_1) = (3, 2)$$

$$(x_2, y_2) = (5, 1)$$

$$\text{Coordinates of P} = \left(\frac{1(5) + 2(3)}{1+2}, \frac{1(1) + 2(2)}{1+2} \right) = \left(\frac{11}{3}, \frac{5}{3} \right)$$

$$\text{Given that P lies on the line } 3x - 18y + k = 0 \Rightarrow 3(11/3) - 18(5/3) + k = 0 \Rightarrow k = 19$$

Q.7. What are the co-ordinates of the fourth vertex if three vertices of a rectangle are the points (3,4), (-1,2), (2,-4)

Answer: Point A(3,4), B(-1,2), C(2,-4), D(x,y)

first divide rectangle in such a way that two triangles are formed

so now in triangle BDC we are going to use section formulae

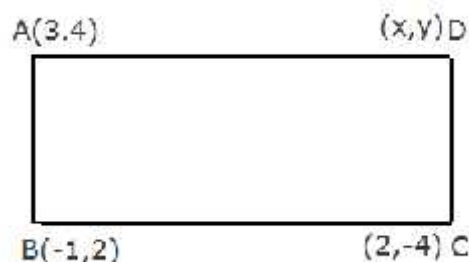
which is $x = [m x_2 + n x_1] / m + n$; $y = [m y_2 + n y_1] / m + n$

So since it is a triangle formed by rectangle so ratio is 2:1

$$\therefore x = \frac{(1)(2) + (2)(-1)}{1+2} \Rightarrow x = \frac{2-2}{3} = 0$$

$$y = \frac{1(-4) + (2)(2)}{3} = 0$$

\therefore The co-ordinate of point D(0,0)



Q8. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, -2) and B(3, 7)

Answer: Let the given line divide the line segment joining the points A(2, -2) and B(3, 7) in a ratio $k : 1$.

$$\text{Coordinates of the point of division} = \left(\frac{3k + 2}{k + 1}, \frac{7k - 2}{k + 1} \right)$$

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This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0 \Rightarrow \frac{6k+4+7k-2-4k-4}{k+1} = 0 \Rightarrow 9k-2=0 \Rightarrow k = \frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, -2) and B(3, 7) is 2:9

Q.9. Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).

Answer : Let O (x, y) be the centre of the circle. And let the points (6, -6), (3, -7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2} \quad OB = \sqrt{(x-3)^2 + (y+7)^2} \quad OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However, $OA = OB$ (Radii of the same circle)

$$\begin{aligned} \Rightarrow \sqrt{(x-6)^2 + (y+6)^2} &= \sqrt{(x-3)^2 + (y+7)^2} \\ \Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y &= x^2 + 9 - 6x + y^2 + 49 + 14y \\ \Rightarrow -6x - 2y + 14 &= 0 \quad \Rightarrow 3x + y = 7 \quad \dots (1) \end{aligned}$$

Similarly, $OA = OC$ (Radii of the same circle)

$$\begin{aligned} \Rightarrow \sqrt{(x-6)^2 + (y+6)^2} &= \sqrt{(x-3)^2 + (y-3)^2} \\ \Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y &= x^2 + 9 - 6x + y^2 + 9 - 6y \\ \Rightarrow -6x + 18y + 54 &= 0 \\ \Rightarrow -3x + 9y &= -27 \quad \dots (2) \end{aligned}$$

On adding equation (1) and (2), we obtain , $10y = -20 \Rightarrow y = -2$

From equation (1), we obtain , $3x - 2 = 7 \Rightarrow 3x = 9 \Rightarrow x = 3$

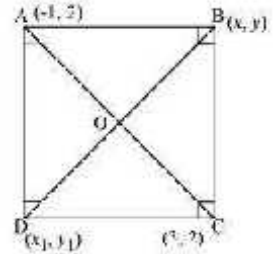
Therefore, the centre of the circle is (3, -2)

Q.10. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Answer: Let ABCD be a square having $(-1, 2)$ and $(3, 2)$ as vertices A and C respectively. Let (x, y) , (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

$$\therefore AB = BC$$



$$\begin{aligned} \Rightarrow \sqrt{(x+1)^2 + (y-2)^2} &= \sqrt{(x-3)^2 + (y-2)^2} \\ \Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 &= x^2 + 9 - 6x + y^2 + 4 - 4y \Rightarrow x = 1 \end{aligned}$$

We know that in a square, all interior angles are of 90° .

In $\triangle ABC$,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \left(\sqrt{(1+1)^2 + (y-2)^2} \right)^2 + \left(\sqrt{(1-3)^2 + (y-2)^2} \right)^2 = \left(\sqrt{(3+1)^2 + (2-2)^2} \right)^2$$

$$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$$

$$\Rightarrow 2y^2 + 16 - 8y = 16 \Rightarrow 2y^2 - 8y = 0 \Rightarrow y(y - 4) = 0$$

\Rightarrow

We know that in a square, the diagonals are of equal length and bisect each other at 90° . Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2} \right) = \left(\frac{1+x_1}{2}, \frac{y+y_1}{2} \right) = (1, 2) \Rightarrow \frac{1+x_1}{2} = 1$$

$$\Rightarrow 1+x_1 = 2 \Rightarrow x_1 = 1 \quad \text{and} \quad \frac{y+y_1}{2} = 2$$

$$y + y_1 = 4 \quad \text{If } y = 0, \quad y_1 = 4$$

$$\text{If } y = 4, \quad y_1 = 0$$

Therefore, the required coordinates are $(1, 0)$ and $(1, 4)$.

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Q.11. The area of the triangle is 5 sq.units, Two of its vertices are (2,1) and (3,-2). The third vertex lies on $y = x + 3$. Find the third vertex

$$\text{Area of } \Delta = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$5 = \frac{1}{2} [2(-1 - x - 3) + 3(x + 3 - 1) + x(1 + 1)]$$

$$10 = 2(-x - 4) + 3(x + 2) + 2x$$

$$10 = -2x - 8 + 3x + 6 + 2x$$

$$10 = 3x - 2$$

$$\Rightarrow x = 4$$

$$\text{and } y = 4 + 3 = 7$$

Therefore, coordinates of third vertex is (4, 7)

Q. 12. What is the value of $x/a + y/b$ if the points (a,0), (0,b), (x,y) are collinear?

Answer: Let A(a, 0), B(0, b) and C(x, y) are the coordinates of the vertices of

ABC. Now the area of ABC is given by,

$$\text{ar}(\Delta ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Now we have,

$$x_1 = a, y_1 = 0; x_2 = 0, y_2 = b; x_3 = x, y_3 = y$$

$$\text{ar}(\Delta ABC) = \frac{1}{2} [a(b - y) + 0(y - 0) + x(0 - b)]$$

$$= \frac{1}{2} [ab - ay + 0 - bx]$$

$$= \frac{1}{2} [ab - ay - bx]$$

Since points A(a, 0), B(0, b) and C(x, y) are collinear, then $\text{ar}(\Delta ABC) = 0$

$$\text{so, } \frac{1}{2} [ab - ay - bx] = 0$$

$$ab - ay - bx = 0$$

$$-ay - bx = -ab$$

$$ay + bx = ab$$

On dividing both sides by ab , we get

$$\frac{y}{b} + \frac{x}{a} = 1$$

$$\text{so, } \frac{x}{a} + \frac{y}{b} = 1$$

Q.13: Find the point on the x – axis which is equidistant from point (2,-5) AND (-2, 9)

Solution:

Let (x , 0) be the point on x-axis which is equidistant from (2, - 5) and (- 2, 9)

therefore, by using distance formula,

$$\begin{aligned}\sqrt{(x-2)^2 + (0+5)^2} &= \sqrt{(x+2)^2 + (0-9)^2} \Rightarrow \sqrt{x^2 + 4 - 4x + 25} = \sqrt{x^2 + 4x + 4 + 81} \\ \Rightarrow \sqrt{x^2 - 4x + 29} &= \sqrt{x^2 + 4x + 85} \Rightarrow x^2 - 4x + 29 = x^2 + 4x + 85 \\ \Rightarrow -4x - 4x &= 85 - 29 \Rightarrow -8x = 56 \Rightarrow x = \frac{-56}{8} \Rightarrow x = -7\end{aligned}$$

Thus, (-7, 0) be the point on x-axis which is equidistant from (2, -5) and (-2, 9).

Q.14. Find the value of k if the points A(k+1, 2k), B(3k, 2k+3) and C (5k – 1, 5k) are collinear.

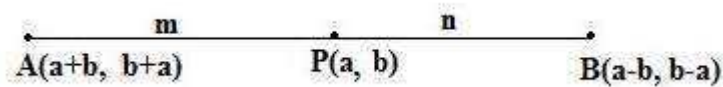
Answer: Since points A, B and C are collinear, we have

Slope of AB = Slope of BC, i.e., $\frac{(2k+3)-2k}{3k-(k+1)} = \frac{5k-(2k+3)}{5k-1-3k}$

$$\frac{3}{2k-1} = \frac{3k-3}{2k-1} \Rightarrow 3k = 6 \Rightarrow k = 2 \text{ Ans.}$$

Q.15. Find the ratio in which the line joining points (a+b , b+a) and (a-b , b-a) is divided by point (a,b).

Ans:



Let the point P(a, b) divided the line joining points A(a+b, b+a) and B(a-b, b-a) in the ratio of m:n.

Therefore, By section formula,

$$a = \frac{m(a-b) + n(a+b)}{m+n} \quad \dots\dots (1) \quad \text{and} \quad b = \frac{m(b-a) + n(b+a)}{m+n} \quad \dots\dots (2)$$

$$\text{From equation (1),} \quad a(m+n) = m(a-b) + n(a+b) \Rightarrow am + an = am - mb + an + bn$$

$$\Rightarrow bm = bn \Rightarrow m = n \quad \text{Therefore, } m:n = n:n = 1:1 \quad \text{Thus, point P divides the line segment AB in 1:1.}$$