## JAIPUR EDUCATION PLUS

## PLOT NO :51, LANE NO:3, Moti Nagar, Queens Road, Jaipur

Q. 1 If the points $(1,-2),(2,3),(-3,2),(-4,-3)$ are the vertices of a parallelogram $A B C D$. Then taking $A B$ as the base find height of the parallelogram.

Ans: Let the vertices of the parallelogram be $\mathrm{A}(1,-2) \mathrm{B}(2,3) \mathrm{C}(-3,2)$ and $D(-4,-3)$.

Join BD to form two triangles $A B D$ and $B C D$.


$$
\begin{aligned}
& \text { Area of a triangle }=\frac{1}{2}\left\{x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right\} \\
& \text { Area of } \triangle A B D=\frac{1}{2}[1(3+3)+2(-3+2)+(-4)(-2-3)] \\
& \quad=\frac{1}{2}[6-2+20] \\
& \quad=\frac{1}{2}[24]=12 \text { square units. } \\
& \text { Area of } \triangle B C D=\frac{1}{2}[2(2+3)+(-3)(-3-3)+(-4)(3-2)] \\
& \quad=\frac{1}{2}[10+18-4] \\
& \quad=\frac{1}{2}[24]=12 \text { square units. }
\end{aligned}
$$

Now,
Area of parallelogram $A B C D=$ Area of $A B D+$ Area of $B C D=12+12=24$ square units.
$A B=2-1^{2}+3+2^{2}=26$
Height $=$ Area/base $=24 / 26$
Q.2. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are $(0,-1),(2,1)$ and $(0,3)$. Find the ratio of this area to the area of the given triangle.

Answer: Let the vertices of the triangle be $\mathrm{A}(0,-1), \mathrm{B}(2,1), \mathrm{C}(0,3)$.

$$
\mathrm{D}=\left(\frac{0+2}{2}, \frac{-1+1}{2}\right)=(1,0) \quad \mathrm{E}=\left(\frac{0+0}{2}, \frac{3-1}{2}\right)=(0,1) \quad \mathrm{F}=\left(\frac{2+0}{2}, \frac{1+3}{2}\right)=(1
$$



Area of a triangle $=\frac{1}{2}\left\{x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right\}$
Area of $\triangle \mathrm{DEF}=\frac{1}{2}\{1(2-1)+1(1-0)+0(0-2)\} \quad=\frac{1}{2}(1+1)=1$ square units
Let $\mathrm{D}, \mathrm{E}, \mathrm{F}$ be the mid-points of the sides of this triangle. Coordinates of $\mathrm{D}, \mathrm{E}$, and $F$ are given by

Area of $\triangle \mathrm{ABC}=\frac{1}{2}[0(1-3)+2\{3-(-1)\}+0(-1-1)]=4$ square units
Therefore, required ratio $=1: 4$

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Q.3. Find the length of the median ADof the triangle ABC whose vertics are $A(7,-3), B(5,3)$ and $C(3,-1)$
,where $D$ is mid-point of the side $B C$
Answer: Since $D$ is the mid point of $B C$, therefore,
$\nu=\left(\frac{3+3}{2} \cdot \frac{3+(-1\rangle}{2}\right)$
$\nu=(4]$,


Now length of AD can be calculated using the distance formula, i.e.,

$$
A D=\sqrt{(4-7)^{2}+[1-(-3)]^{2}}=\sqrt{(-3)^{2}+(4)^{2}} \quad A D=5
$$

The length of the median $A D=5$ units
Q.4. find the ratio in which point $P(-1, y)$ lying on the line segment joining points $A(-3,10)$ and $B(6,-8)$ divides it. Also find the value of $y$.

Solution:
Let the point $(-1, y)$ divides the line segment joining points

$$
A(-3,10) \text { and } B(6,-8) \text { in the ration } k: 1
$$

Then, the coordinate of point $P=\left(\frac{6 k-3}{k+1}, \frac{-8 k+10}{k+1}\right)$
But, coordinate of point $P=(-1, y)$ (given)

Thus,

$$
-1=\frac{6 k-3}{k+1} \quad \Rightarrow-k-1=6 k-3 \quad \Rightarrow k=\frac{2}{7}
$$

$y=\frac{-8 k+10}{k+1} \quad \Rightarrow y=\frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1} \quad \Rightarrow y=6$
Hence, the ratio is 2.7 and $y=6$
Q.5. In what ratio is the line segment joining $A(2,-3)$ and $B(5,6)$ divided by the $x$-axis ? Also, find the coordinates of the point of division.

Answer: Let the line passing through the points $A(2,-3)$ and $B(5,6)$ is

$$
\frac{(y-6)}{(x-5)}=\frac{9}{3} \Rightarrow 3 x-y-9=0
$$

Point where $x$-axis cuts this line can be obtained by putting $y=0$
The required point is $(3,0)$

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Now the point divides the line segment $A B$ in the ratio
$\frac{\sqrt{(5-3)^{2}+(6-0)^{2}}}{\sqrt{(2-3)^{2}}+(-3-0)^{2}}=\frac{\sqrt{4+36}}{\sqrt{1+9}}=\frac{\sqrt{40}}{\sqrt{10}}=\sqrt{4}=\frac{2}{1}$
Q.6. line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point $P$ in ratio $1: 2$ and it lies on the line $3 x-18 y+k=0$

Answer: P divides the line segment joining the points $A(3,2)$ and $B(5,1)$ in the ratio $1: 2$ The coordinates of $P$ can be found put by the section formula
$\left(\frac{m x_{2}+n x_{1}}{m+n}, \frac{m y_{2}+n y_{1}}{m+n}\right)$

Here $m: n=1: 2$
$\left(x_{1}, y_{1}\right)=(3,2)$
$\left(x_{2}, y_{2}\right)=(5,1)$
Coordinates of $\mathrm{P}=\left(\frac{1(5)+2(3)}{1+2}, \frac{1(1)+2(2)}{1+2}\right)=\left(\frac{11}{3}, \frac{5}{3}\right)$

Glven that $P$ lies on the line $3 x-18 y+k=0 \Rightarrow 3(11 / 3)-18(5 / 3)+k=0 \Rightarrow k=19$
Q.7. What are the co-ordinates of the fourth vertex if three vertices of a rectangle are the points $(3,4),(-$ $1,2),(2,-4)$

Answer: Point $A(3.4), B(-1,2), C(2,-4), D(x, y)$ first divide rectangle in such a way that two triangles are formed so now in triangle BDC we are going to use section formulae which is $x=[m x 2+n x 1] / m+n ; y=[m y 2+n y 1] / m+n$

So since it is a triangle formed by rectangle so ratio is $2: 1$
$\therefore x=\begin{gathered}(1)(2)+(2)(-1) \\ 1+2\end{gathered} \Rightarrow x=\frac{2-2}{3}=0$
$\mathrm{y}=\begin{gathered}1 x-4+(2 x 2) \\ 3\end{gathered}$
$\therefore$ The co - ordinate of point $\mathrm{D}(0,0)$

Q8. Determine the ratio in which the line $2 x+y-4=0$ divides the line segment joining the points $A(2,-2)$ and $B(3,7)$

Answer: Let the given line divide the line segment joining the points $A(2,-2)$ and $B(3,7)$ in a ratio $k: 1$.

Coordinates of the point of division $=\frac{3 \mathrm{k}+2}{\mathrm{k}+1}, \frac{7 \mathrm{k}-2}{\mathrm{k}+1}$

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This point also lies on $2 x+y-4=0$

$$
\therefore 2\left(\frac{3 k+2}{k+1}\right)+\left(\frac{7 k-2}{k+1}\right)-4=0 \quad \Rightarrow \frac{6 k+4+7 k-2-4 k-4}{k+1}=0 \quad \Rightarrow 9 k-2=0 \quad \Rightarrow k=\frac{2}{9}
$$

Therefore, the ratio in which the line $2 x+y-4=0$ divides the line segment joining the points $\mathrm{A}(2,-2)$ and $B(3,7)$ is $2: 9$
Q.9. Find the centre of a circle passing through the points $(6,-6),(3,-7)$ and $(3,3)$.

Answer: Let $O(x, y)$ be the centre of the circle. And let the points $(6,-6),(3,-7)$, and $(3,3)$ be representing the points $\mathrm{A}, \mathrm{B}$, and C on the circumference of the circle.

$$
\therefore \mathrm{OA}=\sqrt{(x-6)^{2}+(y+6)^{2}} \quad \mathrm{OB}=\sqrt{(x-3)^{2}+(y+7)^{2}} \quad \mathrm{OC}=\sqrt{(x-3)^{2}+(y-3)^{2}}
$$

However, $\mathrm{OA}=\mathrm{OB}$ (Radii of the same circle)

$$
\begin{align*}
& \Rightarrow \sqrt{(x-6)^{2}+(y+6)^{2}}=\sqrt{(x-3)^{2}+(y+7)^{2}} \\
& \Rightarrow x^{2}+36-12 x+y^{2}+36+12 y=x^{2}+9-6 x+y^{2}+49+14 y \\
& \Rightarrow-6 x-2 y+14=0 \quad \Rightarrow 3 x+y=7 \tag{1}
\end{align*}
$$

Similarly, $\mathrm{OA}=\mathrm{OC} \quad$ (Radii of the same circle)

$$
\begin{align*}
& \Rightarrow \sqrt{(x-6)^{2}+(y+6)^{2}}=\sqrt{(x-3)^{2}+(y-3)^{2}} \\
& \Rightarrow x^{2}+36-12 x+y^{2}+36+12 y=x^{2}+9-6 x+y^{2}+9-6 y \\
& \Rightarrow-6 x+18 y+54=0 \\
& \Rightarrow-3 x+9 y=-27 \tag{2}
\end{align*}
$$

On adding equation (1) and (2), we obtain , 10y $=-20 \Rightarrow \quad y=-2$
From equation (1), we obtain , $3 x-2=7 \Rightarrow 3 x=9 \Rightarrow \quad x=3$
Therefore, the centre of the circle is $(3,-2)$

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Q.10. The two opposite vertices of a square are ( $-1,2$ ) and ( 3,2 ). Find the coordinates of the other two vertices.

Answer: Let ABCD be a square having $(-1,2)$ and $(3,2)$ as vertices $A$ and $C$ respectively. Let $(x, y),\left(x_{1}, y_{1}\right)$ be the coordinate of vertex B and D respectively. We know that the sides of a square are equal to each other.
$\therefore A B=B C$


$$
\begin{aligned}
& \Rightarrow \sqrt{(x+1)^{2}+(y-2)^{2}}=\sqrt{(x-3)^{2}+(y-2)^{2}} \\
& \Rightarrow x^{2}+2 x+1+y^{2}-4 y+4=x^{2}+9-6 x+y^{2}+4-4 y \quad \Rightarrow x=1
\end{aligned}
$$

We know that in a square, all interior angles are of $90^{\circ}$.
In $A B C$,
$A B^{2}+B C^{2}=A C^{2}$
$\Rightarrow\left(\sqrt{(1+1)^{2}+(y-2)^{2}}\right)^{2}+\left(\sqrt{(1-3)^{2}+(y-2)^{2}}\right)^{2}=\left(\sqrt{(3+1)^{2}+(2-2)^{2}}\right)^{2}$
$\Rightarrow 4+y^{2}+4-4 y+4+y^{2}-4 y+4=16$
$\Rightarrow 2 y^{2}+16-8 y=16 \Rightarrow 2 y^{2}-8 y=0 \Rightarrow y(y-4)=0$
$\Rightarrow$
We know that in a square, the diagonals are of equal length and bisect each other at $90^{\circ}$. Let O be the midpoint of AC. Therefore, it will also be the mid-point of BD.

$$
\begin{aligned}
& \text { Coordinate of point } \mathrm{O}=\left(\frac{-1+3}{2}, \frac{2+2}{2}\right) \quad\left(\frac{1+x_{1}}{2}, \frac{y+y_{1}}{2}\right)=(1,2) \Rightarrow \frac{1+x_{1}}{2}=1 \\
& \qquad \begin{array}{ll}
y+y_{1}=4 & \quad \text { If } y=0, \\
\text { If } y=4, \quad x_{1}=2 \Rightarrow x_{1}=1 \quad \text { and } \frac{y+y_{1}}{2}=2
\end{array}
\end{aligned}
$$

Therefore, the required coordinates are $(1,0)$ and $(1,4)$.
Q.11. The area of the triangle is 5 sq.units, Two of its vertices are ( 2,1 ) and ( $3,-2$ ). The third vertex lies on $y$ $=x+3$. Find the third vertex

$$
\begin{aligned}
& \text { Area of } \Delta=\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right] \\
& 5=\frac{1}{2}[2(-1-x-3)+3(x+3-1)+x(1+1)] \\
& 10=2(-x-4)+3(x+2)+2 x \\
& 10=-2 x-8+3 x+6+2 x \\
& 10=3 x-2 \\
& \Rightarrow x=4 \\
& \text { and } y=4+3=7
\end{aligned}
$$

Therefore, coordinates of third vertex is $(4,7)$
Q. 12. What is the value of $x / a+y / b$ if the points $(a, 0),(0, b),(x, y)$ are collinear?

Answer: Let $\mathrm{A}(a, 0), \mathrm{B}(0, b)$ and $\mathrm{C}(x, y)$ are the coordinates of the vertices of
$A B C$. Now the area of $A B C$ is given by,
$\operatorname{ar}(\triangle A B C)=\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$
Now we have,

$$
\begin{aligned}
& x_{1}=a, y_{1}=0 ; x_{2}=0, y_{2}=b ; x_{3}=x, y_{3}=y \\
& \operatorname{ar}(\triangle A B C)=\frac{1}{2}[a(b-y)+0(y-0)+x(0-b)] \\
& =\frac{1}{2}[a b-a y+0-b x] \\
& =\frac{1}{2}[a b-a y-b x]
\end{aligned}
$$

Since points $A(a, 0), B(0, b)$ and $C(x, y)$ are collinear , then $\operatorname{ar}(A B C)=0$

$$
\begin{gathered}
\text { so, } \frac{1}{2}[a b-a y-b x]=0 \\
a b-a y-b x=0 \\
-a y-b x=-a b \\
a y+b x=a b
\end{gathered}
$$

On dividingboth sidesby $a b$, weget

$$
\begin{gathered}
\frac{y}{b}+\frac{x}{a}=1 \\
\text { so, } \frac{x}{a}+\frac{y}{b}=1
\end{gathered}
$$

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Q.13: Find the point on the $x$-axis which is equidistant from point ( $2,-5$ ) AND ( $-2,9$ )

Solution:

Let $(x, 0)$ be the point on $x$-axis which is equidistant from $(2,-5)$ and $(-2,9)$
therefore, by using distance formula,

$$
\begin{aligned}
& \sqrt{(x-2)^{2}+(0+5)^{2}}=\sqrt{(x+2)^{2}+(0-9)^{2}} \Rightarrow \sqrt{x^{2}+4-4 x+25}=\sqrt{x^{2}+4 x+4+81} \\
\Rightarrow & \sqrt{x^{2}-4 x+29}=\sqrt{x^{2}+4 x+85} \quad \Rightarrow x^{2}-4 x+29=x^{2}+4 x+85 \\
\Rightarrow & -4 x-4 x=85-29 \Rightarrow-8 x=56 \Rightarrow x=\frac{-56}{8} \Rightarrow x=-7
\end{aligned}
$$

Thus, $(-7,0)$ be the point on $x$-axis which is equidistant from $(2,-5)$ and $(-2,9)$.
Q.14. Find the value of $k$ if the points $A(k+1,2 k), B(3 k, 2 k+3)$ and $C(5 k-1,5 k)$ are collinear.

Answer: Since points A, B and C are collinear, we have
Slope of $A B=$ Slope of $B C$, i.e., $(2 k+3)-2 k / 3 k-(k+1)=5 k-(2 k+3) / 5 k-1-3 k$
$3 / 2 k-1=3 k-3 / 2 k-1 \Rightarrow 3 k=6 \Rightarrow k=2$ Ans.
Q.15. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by point $(a+b)$.

Ans:


Let the point $P(a, b)$ divided the line joining points $A(a+b, b+a)$ and $B(a-b, b-a)$ in the ratio of $m: n$. Therefore, By section formula,

$$
\begin{equation*}
a=\frac{m(a-b)+n(a+b)}{m+n} \quad \text {......(1) } \quad \text { and } b=\frac{m(b-a)+n(b+a)}{m+n} \tag{2}
\end{equation*}
$$

From equation (1), $\quad a(m+n)=m(a-b)+n(a+b) \quad \Rightarrow a m+a n=a m-m b+a n+b n$
$\Rightarrow b m=b n \quad \Rightarrow m=n \quad$ Therefore, $m: n=n: n=1: 1 \quad$ Thus, point $P$ divides the line segment $A B$ in $1: 1$.

