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PROBLEMS BASED ON CONVERSION OF SOLIDS

1. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylindrical tub is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub (use $\pi = \frac{22}{7}$]

(Ans: 683.83 cm³)

Ans: No. of solid = vol of cone + vol of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 [h + 2r]$$

On substituting we get,

$= 141.17 \text{ cm}^3$

vol of cylinder = $\pi r^2 h$

On substituting we get,

$= 825 \text{ cm}^3$

volume of H_2O left in the cylinder = 825 – 141.17

$= 683.83 \text{ cm}^3$

2. A bucket of height 8 cm and made up of copper sheet is in the form of frustum of right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate
   i) the height of the cone of which the bucket is a part
   ii) the volume of water which can be filled in the bucket
   iii) the area of copper sheet required to make the bucket (Leave the answer in terms of π (Ans: 129 π cm^2)

Ans: Let total height be h

$$\Rightarrow \frac{h}{h+8} = \frac{3}{9} \text{ (similar } \Delta\text{'s)}$$

$\Rightarrow h = 4$ cm

$\therefore$ ht. of cone which bucket is a part = 4 cm

Substitute to get Ans.: for ii) iii)

3. A sphere and a cube have equal surface areas. Show that the ratio of the volume of the sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$.

Ans: S.A. of sphere = S.A of cube

$\Rightarrow 4\pi r^2 = 6a^2$

$\Rightarrow r = \sqrt{\dfrac{6a^2}{4\Pi}}$

$\therefore$ ratio of their volume $\dfrac{v_1}{v_2} = \dfrac{\frac{4}{3}\Pi\gamma^3}{a^3}$

On simplifying & substituting, we get $\sqrt{6} : \sqrt{\pi}$

4. A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed.
(Ans : 3768cu.cm,1318.8 Sq.cm)

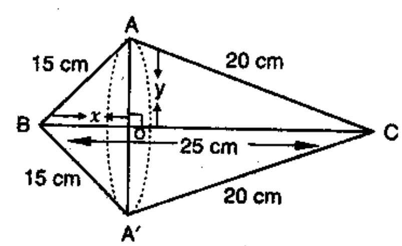
Ans: $BC = \sqrt{15^2 + 20^2} = 25$ cm

Apply Py. Th to right Δ OAB & OAC to get OB = 9cm OA = 12cm

Vol of double cone $= \text{vol of } CAA^1 + \text{vol of } BAA^1$

$= \frac{1}{3}\pi \times 12^2 \times 16 + \frac{1}{3}\pi \times 12^2 \times 9$

$= 3768 \text{ cm}^3$



SA of double cone $= \text{CSA of } CAA^1 + \text{CSA of } BAA^1$

$= \pi \times 12 \times 20 + \pi \times 12 \times 15$

$= 1318.8 \text{ cm}^3$

5. Water in a canal 30 dm wide and 12 dm deep is flowing with a velocity of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is required for irrigation? (Ans: 225000 cu. m)

Ans: Width of canal = 30 dm = 3m
Depth of canal = 1.2 m
Velocity = 10 km / h = 10000 m/h

Length of water column is formed in 30 min = $10000 \times \frac{1}{2} = 5000$ m

Let xm^2 of area be irrigated $\Rightarrow x \times \frac{8}{100} = 5000 \times 1.2 \times 3$

$\Rightarrow x = 225000 \text{ m}^2$

6. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between two vessels is filled with cork dust for heat insulation purposes. How many cubic centimetres of cork dust will be required?
(Ans:1980 cu.cm)

Ans: volume of cork dust required $= \pi R^2 h - \pi r^2 h$
$= \pi\, 42\, [64 - 49]$
$= 1980\ cm^3$

7. An ice-cream cone has a hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, find the volume of ice cream cone. (Ans: $91\frac{2}{3}$ cu.cm)

Ans: Do yourself

8. A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\frac{19}{21}$ cu m of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building.
(Ans : 4m)

Ans: Volume of building $= 41\frac{19}{21}\ m^3$

$\Rightarrow \pi . r^2 . r + \frac{2}{3}\pi r^3 = 41\frac{19}{21}$

$\Rightarrow \pi \times r^3 \times \frac{5}{3} = \frac{880}{21}$

$\Rightarrow r^3 = \frac{880}{21} x \frac{7}{22} x \frac{3}{5}$

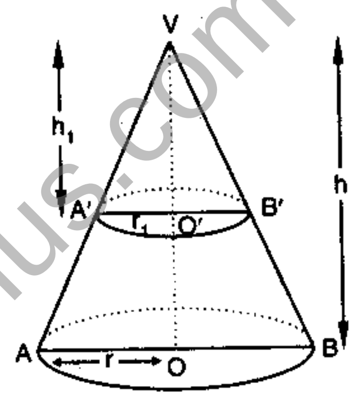
$\Rightarrow r^3 = 8$

$\Rightarrow r = 2\ m$

$\therefore$ height of building $= 4$ cm

9. The height of the Cone is 30 cm A small cone is cut of f at the top by a plane parallel to its base if its volume be $\frac{1}{27}$ of the volume of the given cone at what height above the base is the section cut

(Ans:20 cm)



Ans: $\Delta\, VO^1B \sim \Delta VOB$

$\therefore \frac{H}{h} = \frac{R}{r} = \frac{30}{h} = \frac{R}{r}$ -----(1)

APQ: vol of cone $VA^1B^1 = \frac{1}{27}$ (vol of cone VAB)

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2 H\right)$

$\Rightarrow h^3 = 1000$ (using (1)

h = 10 cm

$\therefore$ height at which section is made (30 – 10) = 20 cm

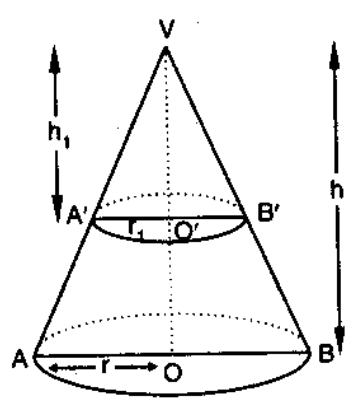
10. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ th of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

(Ans:1:2)

We know that $\Delta\, VO^1B \sim \Delta VOB$

$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$

C. SA of frustum = $\frac{8}{9}$ (CSA of the cone)



$\Pi (R + r)(L - l) = \frac{8}{9}\Pi RL$

$\Rightarrow \left(\frac{R+r}{R}\right)\left(\frac{L-l}{L}\right) = \frac{8}{9}$

$\Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right) = \frac{8}{9}$

$\Rightarrow \left(1+\frac{h}{H}\right)\left(1-\frac{h}{H}\right) = \frac{8}{9}$

On simplifying we get $\frac{h^2}{H^2} = \frac{1}{9}$

$$\frac{h}{H} = \frac{1}{3}$$

$\Rightarrow$ H = 3h

required ratios = $\frac{h}{H-h} = \frac{1}{2}$

11. Two right circular cones X and Y are made X having 3 times the radius of Y and Y having half the Volume of X. Calculate the ratio of heights of X and Y. (Ans: 9 : 2)

Ans: Let radius of cone X = r

Radius of Cone Y = 3r

V of Y = $\frac{1}{2}$ volume of X

$\frac{1}{3}\pi r^2{}_1 h_1 = \frac{1}{2}\left(\frac{1}{3}\pi r^2{}_2 h_2\right)$

=> $r^2 h_1 = \frac{1}{2}\, 9 r^2 h_2$

$\frac{h_1}{h_2} = \frac{9r^2}{2r^2}$

$\frac{h_1}{h_2} = \frac{9}{2}$

12. If the areas of three adjacent faces of cuboid are x, y, z respectively, Find the volume of the cuboids.

Ans: lb = x , bh = y, hl = z

Volume of cuboid = lbh

$V^2 = l^2 b^2 h^2 = xyz$

$V = \sqrt{xyz}$

13. A shuttlecock used for playing badminton has the shape of a frustum of a Cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, and the height of the entire shuttlecock is 7cm. Find the external surface area.

(Ans: $74.26cm^2$)

Ans: r_1 = radius of lower end of frustum = 1 cm

r_2 = radius of upper end = 2.5 cm

h = ht of frustum = 6cm

$l = \sqrt{h^2 + (r_2 - r_1)^2}$ = 6.18 cm

External surface area of shuttlecock = $\pi (r_1 + r_2)\, l + 2\pi r^2{}_1$

On substituting we get, = $74.26\ cm^2$

14. A Solid toy in the form of a hemisphere surmounted by the right circular cone of height 2cm and diameter of the base 4 cm .If a right circular cylinder circumscribes the toy, find how much more space than the toy it will cover.

(Ans: 8π)

Ans : Self practice

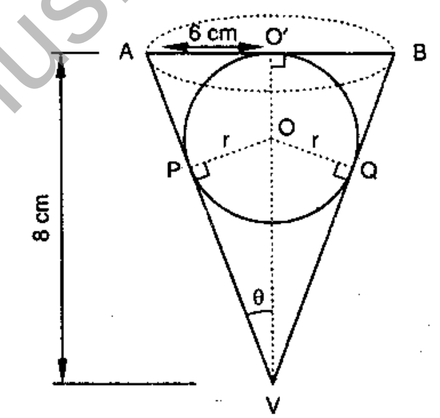
15. A conical vessel of radius 6cm and height 8cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in the figure. What fraction of water flows out.

[Ans: $\frac{3}{8}$]

Ans: This problem can be done in many ways

Let "r" be the radius of sphere

In right triangle



Tan θ = $\frac{6}{8} = \frac{3}{4}$

=> Sin θ = $\frac{3}{5}$

in rt Δ

Sin θ = $\frac{r}{VO} = \frac{3}{5} = \frac{r}{8-r}$

r = 3 cm

Volume of H_2O that flows out of cone = volume of sphere

fraction of water Overflows = $\frac{\text{volume f sphere}}{\text{Volume of cone}}$

$= \frac{36\pi}{96\pi} = \frac{3}{8}$

16. A golf ball has a diameter equal to 4.1cm. Its surface has 150 dimples each of radius 2mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.

(Ans: 71.68)

Ans: SA of ball = $4\pi \times (\frac{4.1}{2})^2 = 16.8\,\pi\text{ cm}^2$

TSA exposed to surroundings

$= \text{SA of ball} - 150 \times \pi r^2 + 150 \times 2\pi r^2$

$= 16.8\,\pi + 150\,\pi r^2$

$= 71.68\text{ cm}^2$

17. A solid metallic circular cone 20cm height with vertical angle 60 is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm Find the length of the wire. (Ans:7964.4m)

Ans: Let r_2 & r_2 be the two ends of the frustum $\frac{r_1}{20} = \tan 30$

$$r_1 = \frac{20}{\sqrt{3}}; r_2 \frac{10}{\sqrt{3}} cm$$

volume of frustum = $\frac{1}{3}\pi\ h\ (r^2{}_1 + r^2{}_2 + r_1 r_2)$

$$= \frac{1}{3}\pi \times 10 \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) cm$$

Since the frustum is drawn into a wire of length x

Volume of frustum = volume of cylinder

$$\frac{1}{3}\pi \times 10 \times \frac{700}{3} = \pi\left(\frac{1}{32}\right)^2 \times x$$

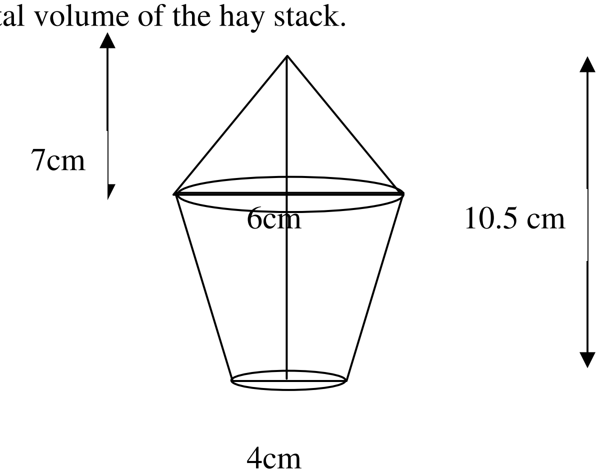
$$\Rightarrow x = \frac{7168000}{9} \text{ cm}$$

$x = 7964.4\text{m}$

18. If the areas of the circular bases of a frustum of a cone are $4cm^2$ and $9cm^2$ respectively and the height of the frustum is 12cm. What is the volume of the frustum. (Ans:$44cm^2$).

Ans: Self practice

19. The lower portion of a hay stack is an inverted cone frustum and the upper part is a cone find the total volume of the hay stack. (Ans: 135.67cu cm)



Ans: Self practice

20. A vessel in shape of a inverted cone is surmounted by a cylinder has a common radius of 7cm this was filled with liquid till it covered one third the height of the cylinder. If the height of each part is 9cm and the vessel is turned upside down. Find the volume of the liquid and to what height will it reach in the cylindrical part. (Ans:924 π cu cm, 6cm)

Ans: Volume of liquid in the vessel = $\frac{1}{3}\pi\ (7)^2\ (9) + \pi\ (7)^2\ (3)$

$= 924$ cu cm

height of cylindrical part = $\dfrac{924}{\frac{22}{7}\times 49}$ = 6 cm