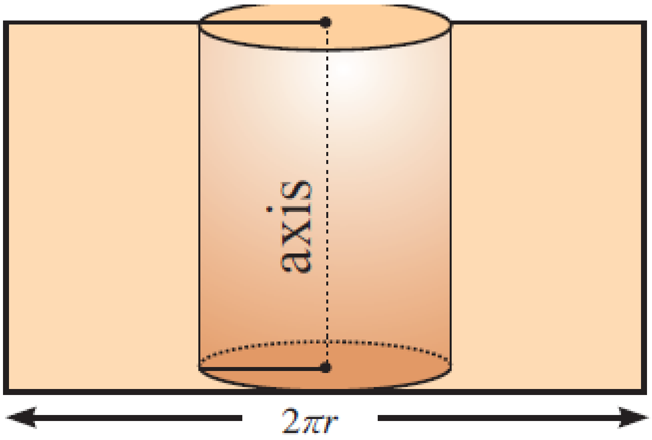
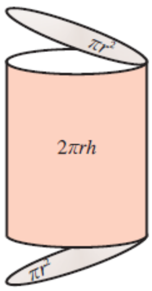
Curved Surface area of a solid right circular cylinder



Curved Surface Area of a cylinder, CSA = Circumference of the base $\times$ Height $= 2\pi r \times h$
$= 2\pi rh$ sq. units.

Total Surface Area of a solid right circular cylinder



Total Surface Area, TSA = Area of the Curved Surface Area + 2 $\times$ Base Area

$= 2\pi rh + 2 \times \pi r^2$

Thus, TSA $= 2\pi r(h + r)$ sq.units.

Right circular hollow cylinder

we have, curved surface area, CSA = External surface area + Internal surface area

$= 2\pi Rh + 2\pi rh$

Thus, CSA $= 2\pi h(R + r)$ sq.units

Total surface area, TSA = CSA + 2 $\times$ Base area

$= 2\pi h(R + r) + 2 \times [\pi R^2 - \pi r^2]$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

$\therefore$ TSA $= 2\pi(R + r)(R - r + h)$ sq.units.

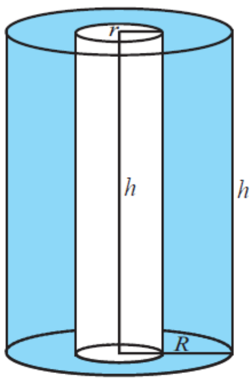


Fig. 8.8

Remark

Thickness of the hollow cylinder, w = $R - r$.

Curved surface area of a hollow cone

Let us consider a sector with radius l and central angle $\theta°$. Let L denote the length of the arc. Thus, $\frac{2\pi l}{L} = \frac{360^\circ}{\theta^\circ}$

$$\implies L = 2\pi l \times \frac{\theta^\circ}{360^\circ} \qquad (1)$$

Now, join the radii of the sector to obtain a right circular cone.

Let r be the radius of the cone.

Hence, $L = 2\pi r$

From (1) we obtain,

$$2\pi r = 2\pi l \times \frac{\theta^\circ}{360^\circ}$$

$$\implies r = l\left(\frac{\theta^\circ}{360^\circ}\right)$$

$$\implies \frac{r}{l} = \left(\frac{\theta^\circ}{360^\circ}\right)$$

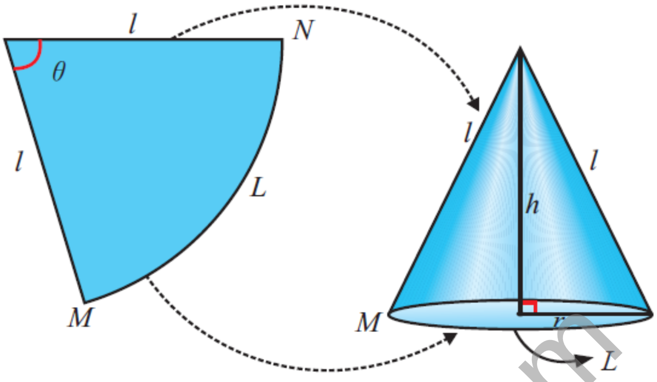


Fig. 8.16

Remarks

When a sector of a circle is folded into a cone, the following conversions are taking place:

Sector	Cone
Radius (l) →	Slant height (l)
Arc Length (L) →	Perimeter of the base $2\pi r$
Area →	Curved Surface Area πrl

Let A be the area of the sector. Then

$$\frac{\pi l^2}{A} = \frac{360^\circ}{\theta^\circ} \qquad (2)$$

Then the curved surface area of the cone $\}$ = Area of the sector

Thus, the area of the curved surface of the cone $\}$ $A = \pi l^2\left(\frac{\theta^\circ}{360^\circ}\right) = \pi l^2\left(\frac{r}{l}\right)$.

Hence, the curved surface area of the cone $= \pi rl$ sq.units.

(ii) Total surface area of the solid right circular cone

Total surface area of the solid cone = $\{$ Curved surface area of the cone + Area of the base

$$= \pi rl + \pi r^2$$

Total surface area of the solid cone $= \pi r(l + r)$ sq.units.

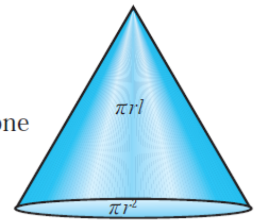
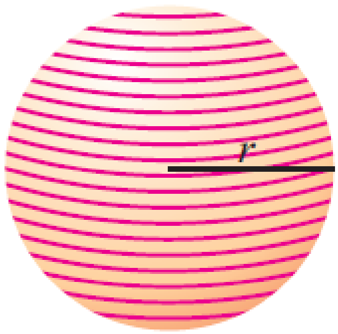


Fig. 8.17

Curved surface area of a solid sphere



Take a circular disc, paste a string along a diameter of the disc and rotate it 360°. The object so created looks like a ball. The new solid is called sphere.

The following activity may help us to visualise the surface area of a sphere as four times the area of the circle with the same radius.

- Take a plastic ball.
- Fix a pin at the top of the ball.
- Wind a uniform thread over the ball so as to cover the whole curved surface area.
- Unwind the thread and measure the length of the thread used.
- Cut the thread into four equal parts.
- Place the strings as shown in the figures.

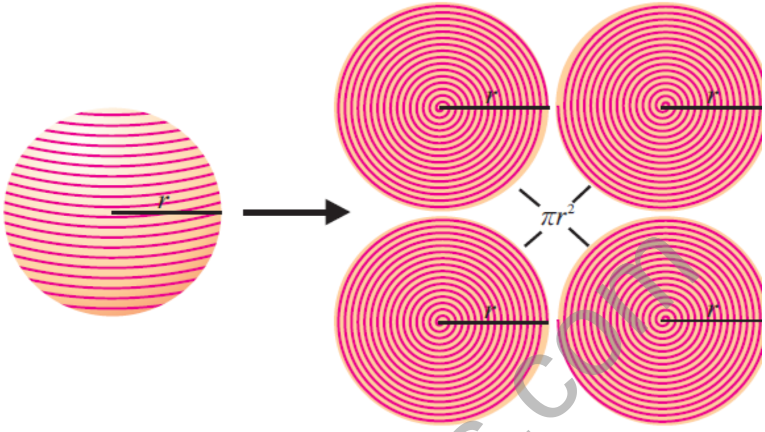


Fig. 8.21

- Measure the radius of the sphere and the circles formed.

Now, the radius of the sphere = radius of the four equal circles.

Thus, curved surface area of the sphere, CSA = $4 \times$ Area of the circle = $4 \times \pi r^2$

$\therefore$ The curved surface area of a sphere = $4\pi r^2$ sq. units.

(ii) Solid hemisphere

A plane passing through the centre of a solid sphere divides the sphere into two equal parts. Each part of the sphere is called a solid hemisphere.

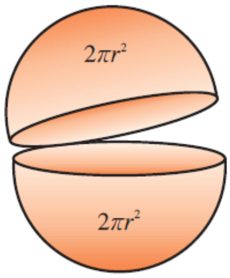


Fig. 8.22

$$\text{Curved surface area of a hemisphere} = \frac{\text{CSA of the Sphere}}{2}$$

$$= \frac{4\pi r^2}{2} = 2\pi r^2 \text{ sq.units.}$$

Total surface area of a hemisphere, TSA = Curved Surface Area + Area of the base Circle

$$= 2\pi r^2 + \pi r^2$$

$$= 3\pi r^2 \text{ sq.units.}$$

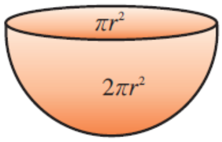


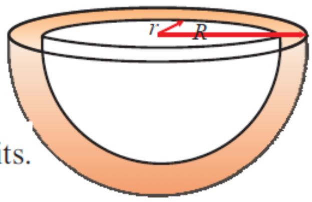
Fig. 8.23

(iii) Hollow hemisphere

Let R and r be the outer and inner radii of the hollow hemisphere.

Now, its curved surface area = Outer surface area + Inner surface area $= 2\pi R^2 + 2\pi r^2$

$$= 2\pi(R^2 + r^2) \text{ sq.units}.$$



$$\text{The total surface area} = \begin{cases} \text{Outer surface area + Inner surface area} \\ + \text{ Area at the base} \end{cases}$$

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) = 2\pi(R^2 + r^2) + \pi(R + r)(R - r) \text{ sq.units.}$$

Volume of a right circular cylinder

(i) Volume of a solid right circular cylinder

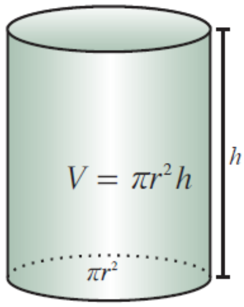


Fig. 8.28

The volume of a solid right circular cylinder is the product of the base area and height.

That is, the volume of the cylinder, V = Area of the base $\times$ height

$= \pi r^2 \times h$

Thus, the volume of a cylinder, $V = \pi r^2 h$ cu. units.

(ii) Volume of a hollow cylinder

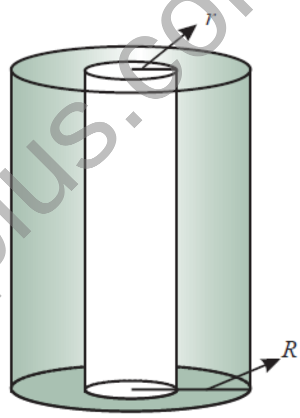


Fig. 8.29

Let R and r be the external and internal radii of a hollow right circular cylinder respectively. Let h be its height.

Then, the volume, $V = \begin{Bmatrix}\text{Volume of the}\\ \text{outer cylinder}\end{Bmatrix} - \begin{Bmatrix}\text{Volume of the}\\ \text{inner cylinder}\end{Bmatrix}$

$= \pi R^2 h - \pi r^2 h$

Hence, the volume of a hollow cylinder,

$V = \pi h (R^2 - r^2)$ cu. units.

Volume of a right circular cone

Let r and h be the base radius and the height of a right circular cone respectively.

The volume V of the cone is given by the formula: $V = \frac{1}{3} \times \pi r^2 h$ cu. units.

$3 \times$ (Volume of the cone) = Volume of the cylinder $= \pi r^2 h$

Volume of a Frustum of a Cone

The volume of a frustum of a cone is nothing but the difference between volumes of two right circular cones. (See Fig. 8.35) Consider a frustum of a solid right circular cone.

Let R be the radius of the given cone. Let r and x be the radius and the height of the smaller cone obtained after removal of the frustum from the given cone.

Let h be the height of the frustum.

Now, $\left.\begin{matrix}\text{the volume of the}\\ \text{frustum of the cone}\end{matrix}\right\}$, $V = \begin{Bmatrix}\text{Volume of the}\\ \text{given cone}\end{Bmatrix} - \begin{Bmatrix}\text{Volume of the}\\ \text{smaller cone}\end{Bmatrix}$

$= \frac{1}{3} \times \pi \times R^2 \times (x + h) - \frac{1}{3} \times \pi \times r^2 \times x$

Thus, $V = \frac{1}{3}\pi[x(R^2 - r^2) + R^2 h]$. (1)

From the Fig. 8.36 we see that $\Delta BFE \sim \Delta DGE$

$$\therefore \quad \frac{BF}{DG} = \frac{FE}{GE}$$

$$\implies \frac{R}{r} = \frac{x+h}{x}$$

$$\implies Rx - rx = rh$$

$$\implies x(R-r) = rh$$

Thus, we get $\quad x = \frac{rh}{R-r} \qquad (2)$

Now, (1) $\implies V = \frac{1}{3}\pi[x(R^2 - r^2) + R^2 h]$

$$\implies = \frac{1}{3}\pi[x(R-r)(R+r) + R^2 h]$$

$$\implies = \frac{1}{3}\pi[rh(R+r) + R^2 h] \text{ using (2)}$$

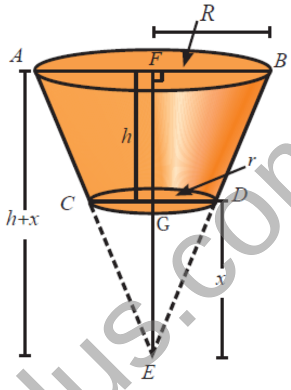


Fig. 8.36

Hence, the volume of the frustum of the cone,

$$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr) \text{ cu. units.}$$

* Curved surface area of a frustum of a cone $= \pi(R+r)l$ where $l = \sqrt{h^2 + (R^2 - r^2)}$
* Total surface area of a frustum of a the cone $= \pi l(R+r) + \pi R^2 + \pi r^2, l = \sqrt{h^2 + (R^2 - r^2)}$

Volume of a Solid Sphere

The following simple experiment justifies the formula for volume of a sphere,

$$V = \frac{4}{3}\pi r^3 \text{ cu.units.}$$

Take a cylindrical shaped container of radius R and height h. Fill the container with water. Immerse a solid sphere of radius r, where $R > r$, in the container and measure the quantity of the displaced water. Now, the volume of the solid sphere is same as that of the displaced water.

Thus, the volume of the sphere,

$$V = \frac{4}{3}\pi r^3 \text{ cu.units.}$$

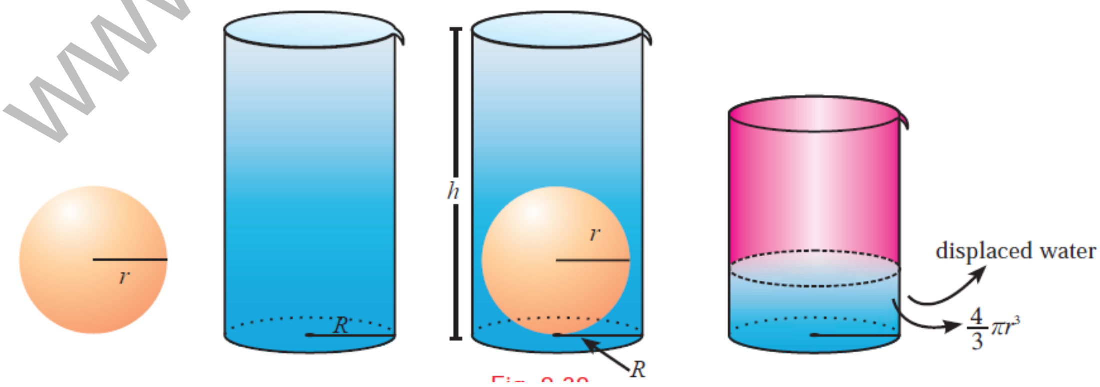


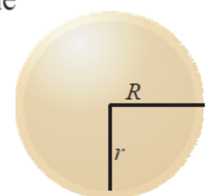
Fig. 8.38

Volume of a hollow sphere

If the inner and outer radius of a hollow sphere are r and R respectively, then

$$\left.\begin{matrix}\text{Volume of the}\\ \text{hollow sphere}\end{matrix}\right\} = \left.\begin{matrix}\text{Volume of the}\\ \text{outer sphere}\end{matrix}\right\} - \left\{\begin{matrix}\text{Volume of the}\\ \text{inner sphere}\end{matrix}\right.$$

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$



$\therefore$ Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$ cu. units.

Volume of a solid hemisphere

Volume of the solid hemisphere $= \frac{1}{2} \times$ volume of the sphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3 \text{ cu.units.}$$

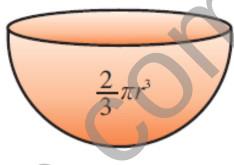


Fig. 8.40

Volume of a hollow hemisphere

$$\left.\begin{matrix}\text{Volume of a hollow}\\ \text{hemisphere}\end{matrix}\right\} = \left.\begin{matrix}\text{Volume of outer}\\ \text{hemisphere}\end{matrix}\right\} - \left\{\begin{matrix}\text{Volume of inner}\\ \text{hemisphere}\end{matrix}\right.$$

$$= \frac{2}{3} \times \pi \times R^3 - \frac{2}{3} \times \pi \times r^3$$

$$= \frac{2}{3}\pi(R^3 - r^3) \text{ cu.units.}$$

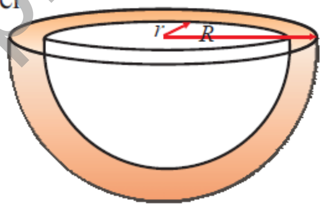


Fig. 8.41