Periodic Motion .

Any motion, which repeats itself at regular interval of time, is called a periodic motion.

Eg: 1) Rotation of earth around sun. 2) Vibrations of a simple pendulum. 3) Rotation of electron around nucleus.

Oscillatory motion:

If the periodic motion takes place to and fro along a straight line or along the arc of a circle, the motion is said to be an oscillatory motion.

Eg: 1) Oscillations of a simple pendulum. 2) Vibrations of the prongs of a tuning fork.

Simple Harmonic Motion.

Let a particle be displaced through a distance 'y' from mean position O. Then a restoring force F tends to bring the particle back to O, due to the property of elasticity.

If y is small, then the force F is proportional to y and opposes the increase of y.

$\therefore$ F = - k y. $\rightarrow (1)$; where k is a constant called force constant, having unit N/m.

So *simple harmonic motion is the motion in which the restoring force is proportional to displacement from the mean position and opposes its increase*.

Acceleration 'a' of a body vibrating in SHM is given by $a = \dfrac{d^2y}{dt^2}$

$$\therefore \text{ Restoring force, } F = m\frac{d^2y}{dt^2} \rightarrow (2)$$

From (1) and (2), $m\dfrac{d^2y}{dt^2} = -ky$

$$\frac{d^2y}{dt^2} = -\frac{k}{m}y$$

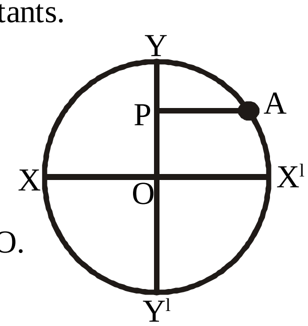
$$\text{i.e. } \frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

$$\text{Also, } \underline{\underline{\frac{d^2y}{dt^2} + \omega^2 y = 0}} \;;\; \text{where } \omega = \sqrt{\frac{k}{m}}$$

This is the differential equation of SHM.

The solution of this equation is $y = A\sin(\omega t + \phi)$. ; where A, ω and ϕ are constants.

Relation between SHM and uniform circular motion.



Consider a particle A undergoing uniform circular motion in a circle having XOX^1 and YOY^1 as horizontal and vertical diameters.

Let P be the foot of the perpendicular drawn from A to the vertical diameter. Then P is called the **projection of A or the shadow of A** when A is at O.

Now as A moves from X to Y, P moves from O to Y along the vertical diameter. As A moves from Y to X^1, P comes back from Y to O. Thus as A completes its journey along the circumference of the circle, its projection moves from O to Y, Y to O, O to Y^1 and Y^1 to O. If the particle A keeps on moving continuously in uniform circular motion, its projection P keeps on vibrating to and fro about O. Motion of P along YOY^1 is simple harmonic.

Simple harmonic motion is defined as the projection of uniform circular motion on the diameter of circle of reference.

Here the center O of circle is the mean position.

**** Another definition for SHM**

We know, for SHM, $\frac{d^2y}{dt^2} = -\frac{k}{m}y$ Here the -ve sign indicates that the acceleration is always directed towards the mean position so that it will oppose the increase in displacement.

A particle is said to move in SHM if its acceleration is proportional to displacement and is always directed towards the mean position.

**** Characteristics of SHM**

1) **Displacement (y) and Amplitude (A)**

Displacement of a particle vibrating in SHM at any instant is defined as its distance from the mean position at that instant.

The displacement of a particle executing SHM is given by $y = A\sin wt$, where $\omega = \frac{\theta}{t} = \frac{2\pi}{T}$

Here y is maximum when $\sin wt$ is maximum. The extreme values of $\sin\omega t = \pm 1 \quad \therefore\ y = \pm A$; where **A is called amplitude of vibrations.**

Amplitude of a particle executing SHM is defined as its maximum displacement on either side of the mean position.

2) Velocity (v)

$$\text{Velocity,}\quad v = \frac{dy}{dt} = \frac{d}{dt}(A\sin\omega t)$$

$$v = A\omega\cos\omega t$$

$$\text{i.e,}\ v = A\omega\sqrt{1-\sin^2\omega t}$$

$$v = A\omega\sqrt{1-\frac{y^2}{A^2}}$$

$$\therefore\ v = A\omega\sqrt{\frac{A^2-y^2}{A^2}}$$

$$\underline{\underline{v = \omega\sqrt{A^2-y^2}}}$$

From this equation, it is clear that the particle will have maximum velocity at $y = 0$.

$\therefore$ maximum velocity $v = A\omega$.

At extreme positions, $v = \omega\sqrt{A^2 - A^2} = 0$.

Therefore, a particle vibrating in SHM passes with maximum velocity through the mean position and is at rest at the extreme positions.

3) **Acceleration**

We have, $y = A\sin\omega t$.

Acceleration, $a = \frac{d^2y}{dt^2} = -A\omega^2\sin\omega t$.

ie $a = -\omega^2 y$.

At mean position, $y = 0 \qquad \therefore\ a = 0$.

At extreme position, $y = A \qquad \therefore\ a = w^2A$. (maximum)

4) **Time period (T)**

It is the time taken by the particle to complete one vibration.

If ω is the angular velocity of the particle, $T = \frac{2\pi}{\omega}$.

But acceleration $= -\omega^2$ (displacement)

$$\therefore \omega^2 = \frac{\text{acceleration}}{\text{displacement}} \qquad \therefore \omega = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Also $\omega = \sqrt{\frac{k}{m}}$ $\qquad$ $\therefore T = 2\pi\sqrt{\frac{m}{k}}$

**** Important terms connected with SHM**

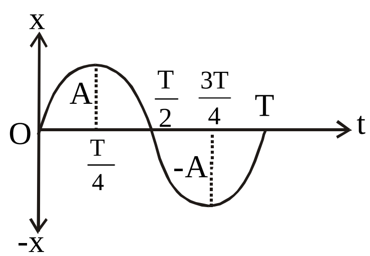
1) Wave length (λ) It is the distance travelled by a wave by the time the particle of the medium completes one vibration.

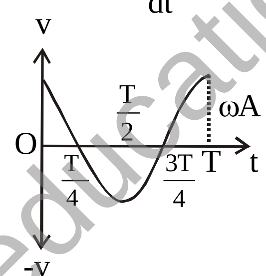
2) Frequency (ν) It is the number of vibrations made by a particle in one second.

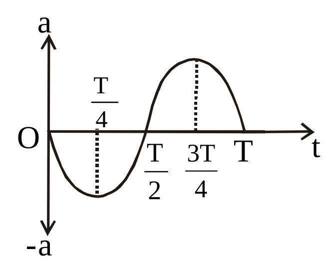
Note: Relation between frequency (ν) and wavelength (λ) is $v = \nu\lambda$; where v is the velocity of the wave.

Graphical representation of displacement, velocity and acceleration.

Displacement $x = A\sin\omega t$ $\qquad$ $v = \frac{dx}{dt}$ $A\omega\cos\omega t$ $\qquad$ $a = -\omega^2 A\sin\omega t$







****NB Energy of SHM**

A particle executing SHM has two types of energy – potential and kinetic.

1) **Potential Energy.**

Potential energy is defined as the work done in displacing a particle from its equilibrium position.

Let a particle be displaced through a distance y from mean position. Then restoring force, $F = -ky$; k is the force constant. Now if we displace the particle further through a distance dy;

Then work done, $dw = -F\,dy = k\,y\,dy$.

$\therefore$ The total work done when the particle is displaced from 0 to y; $W = \int_0^y k\,y\,dy = \frac{1}{2}ky^2$

$\therefore$ Potential Energy; $E_P = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2y^2$ $\qquad$ [since $k = m\omega^2$. because $\omega^2 = \frac{k}{m}$]

At mean position, $y = 0$. $\quad \therefore E_p = 0$.

At extreme position $y = A$ $\quad \therefore E_P$ $\frac{1}{2}m\omega^2A^2$ (maximum).

2) **Kinetic Energy.**

It is the energy possessed by a particle by virtue of its motion. Consider a particle of mass m executing SHM with angular frequency ω.

The velocity of particle, $v = \omega\sqrt{A^2 - y^2}$

Kinetic energy, $E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - y^2\right) = \frac{1}{2}k(A^2 - y^2)$

At mean position $x = 0$. $\therefore E_K = \frac{1}{2}m\omega^2 A^2$ maximum

At extreme position $x = A$ $\therefore E_K = 0$.

3) **Total energy of a particle executing SHM**.

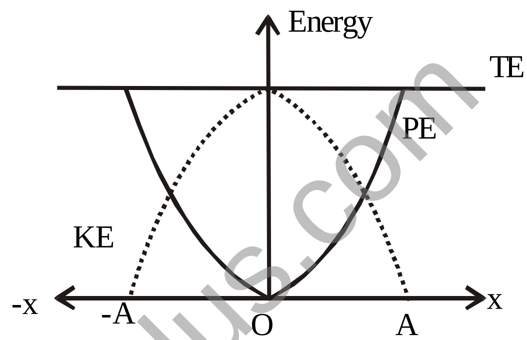
Total Energy, E = Potential Energy + Kinetic Energy.

$$E = \frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$$

Also $E = \frac{1}{2}kA^2$

Since k and A are constant for SHM, total energy of a particle executing SHM is constant.

Graphical representation of energy of SHM is as shown:



**NB Applications of SHM.

NB 1) **Simple Pendulum

A simple pendulum consists of a heavy metallic bob suspended from a fixed point by means of a light inextensible string.

Consider a pendulum of length ℓ oscillating to and fro about its equilibrium position PO. Let θ be the angular displacement of the pendulum at any instant t. At this instant, the bob of the pendulum is at C.

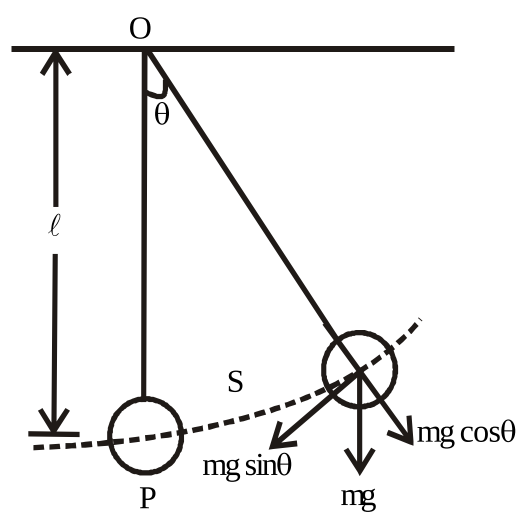
Here the weight of the bob mg is acting vertically downwards. mg can be resolved into two components (1) mg cosθ acting along string and (2) mg sinθ acting perpendicular to the string.

The component mg sinθ is acting towards the mean position and tries to bring the pendulum to its mean position. This is the restoring force acting on the pendulum.

Also the component mg cosθ is balanced by the tension T of the string.

$\therefore$ Restoring force, $F = mg\sin\theta$.

$T = mg\cos\theta$.



Now $F = mg\sin\theta$.

For small amplitudes, $\sin\theta \approx \theta$.

$\therefore F = mg\theta$

But angle $\theta = \frac{\text{arc}}{\text{radius}} = \frac{OC}{PC} = \frac{S}{\ell}$; where S = OC

$\therefore$ Restoring force, $F = mg\theta = \frac{mgS}{\ell}$

Acceleration of the bob towards mean position,

$$a = \frac{\text{force}}{\text{mass}} = \frac{mgS}{\ell m} = \frac{g}{\ell}S$$

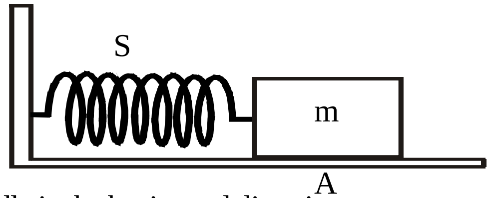
$$\therefore a = \omega^2 S \text{ ; where } \omega = \sqrt{\frac{g}{\ell}}$$

Since ω is constant, $a \propto S$. Hence the motion is simple harmonic.

$$\therefore \text{ Period of oscillation, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

Hence period of oscillation of a simple pendulum, $T = 2\pi\sqrt{\frac{\ell}{g}}$

Note: *A simple pendulum whose period of oscillation is two seconds is called a seconds pendulum*.



NB **Oscillations of a spring.

Consider a spring S whose one end is fixed to a rigid support and the other end attached to a mass m. In equilibrium position, the mass is at A. When the mass is displaced towards right and left, it oscillates simple harmonically in the horizontal direction.

According to Newton's second law, Force = $= m\frac{d^2y}{dt^2} = -ky$, – ve sign shows that force is directed upwards.

$$\therefore\ m\frac{d^2y}{dt^2} + ky = 0. \qquad \text{Or} \qquad \frac{d^2y}{dt^2} + \frac{k}{m}y = 0 \ \ldots\ldots\ldots(1)$$

This is similar to equation of simple harmonic motion, $\frac{d^2y}{dt^2} + \omega^2 y = 0$ ....(2)

Comparing (1) and (2) , $\omega = \sqrt{\frac{k}{m}}$

Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Knowing the values of m and k, the value of T can be calculated.

Note: *Spring constant or force constant of a spring is the force required to produce unit elongation in the spring.*

Special Cases:

a) **Springs connected in parallel.**

If two springs S_1 and S_2 having spring constants k_1 and k_2 are connected in parallel supporting a mass m , then force constant of the combination, $k = k_1 + k_2$.

$\therefore$ Time period of oscillation, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$. If $k_1 = k_2 = k$, $T = 2\pi\sqrt{\frac{m}{2k}}$

b) Springs connected in series:

If two springs S_1 and S_2 having spring constants k_1 and k_2 are connected in series supporting a mass m, then spring constant, $k = \frac{k_1 k_2}{k_1 + k_2}$

Period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$

NB **Free and Damped Oscillator.

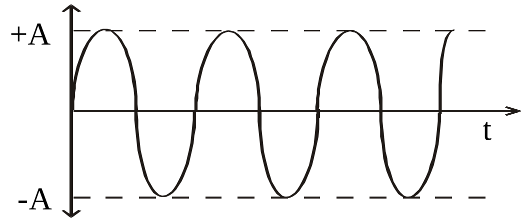
An oscillator oscillating without the influence of an external force is called free oscillator or undamped oscillator.

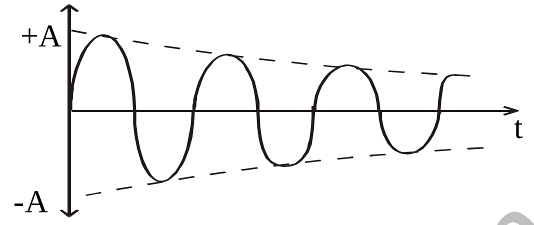
Here the amplitude of oscillation and its total energy remains constant.

In actual practice, all oscillators are subjected to frictional forces of one kind or the other. A real oscillator must work against the dissipative forces. This leads to decrease in energy of the oscillator. Since energy is proportional to the square of amplitude, of vibration of a real oscillator gradually decreases with time.

The oscillation of a system with progressively decreasing amplitude due to the presence of friction is called a damped oscillator.

The free and damped oscillations are represented graphically as shown:





NB **Forced Oscillations and Resonance

Let an external periodic force be applied on a body. The frequency of this external force need not necessarily be same as the natural frequency of the body. The body now begins to vibrate with its natural frequency. But soon the natural vibrations of the body die out and the body continues to vibrate with the frequency of the impressed driving force. These vibrations continue as long as the external periodic force is operative. Such oscillations of the body are called *forced or driven oscillations*.

The oscillations produced by an oscillator due to the application of an external periodic force with a frequency different from the natural frequency of the body are called forced or driven oscillations.

As a special case it is possible that the frequency of driving force may coincide with the natural frequency of driving body. In that case, the body is said to oscillate in **resonance** with the external force. Here the amplitude of vibration is extremely large.

Resonance is the phenomenon of setting a body into vibrations by a strong periodic force whose frequency coincides with the natural frequency of the body.

Examples of resonance:

1) A vibrating tuning fork when placed near the mouth of a particular length of air column produces a loud sound due to resonance.

2) Army while crossing a suspension bridge breaks its steps. This is to avoid resonance between bridge and the impressed force of their feet. Otherwise the bridge may collapse due to large amplitude in case of resonance.

*** The Tacoma Bridge in Washington was opened in 1940. Four months after the opening winds produced a fluctuating resultant force in resonance with the natural frequency of the bridge. This increased the amplitude of oscillation steadily and finally the bridge was destroyed.*

*** Buildings and Earthquake.*

....

WAVES

****NB Waves:**

A wave is a disturbance that propagates energy from one place to other without the transportation of matter.

Eg: Ripples formed on the surface of water when a stone is thrown.

Waves, which do not require a medium for its propagation are called non – mechanical *waves.*

Eg: light wave, heat wave etc.

Waves, which require material medium for their propagation, are called mechanical waves.

Eg: Waves on strings, sound waves etc.

Waves are generally classified into two — **longitudinal waves and transverse waves.**

(a) **Longitudinal waves**:

If the particles of the medium vibrate parallel to the direction of propagation of the wave, it is called a longitudinal wave.

Consider a tuning fork vibrating in air. When the fork vibrates, each of its prongs move to either sides. When a prong moves, say to right, the layer of air in contact with it is pushed to the right. Thus a region of increased pressure is formed. This is called a **condensation**. The prong then moves to the left. Then there will be a lowering of pressure on the right side of the prong. Thus a region of reduced pressure is formed. This is called **rarefaction**. So as the fork vibrates, condensations and rarefactions are produced alternatively and the wave propagates. Eg: sound waves.

(b) **Transverse waves**

If the particles of the medium vibrate perpendicular to the direction of propagation of the wave, it is called a transverse wave.

When a stone is thrown into water, transverse waves are formed. As transverse wave propagates through the medium, some portions of the medium get raised above its normal level and are called **crests**; while some other portions get depressed below the normal level and are called **troughs**.

Eg: Waves produced in a stretched string and light waves are transverse waves.

Important terms connected with wave motion.

a) **Amplitude** (A) *of a wave is the maximum displacement of any particle of the medium from its mean position.*

b) **Frequency** (ν) *of a wave is the number of vibrations made by a particle of the medium per second.* Unit is hertz (Hz.)

c) **Period** (T) *of a wave is the time required by a particle to make one complete vibration*. If ν is the frequency of a wave, its period, $T = \frac{1}{\nu}$ seconds.

d) **Wavelength** (λ): of a wave is *the distance traveled by the wave by the time a particle completes one vibration.*

** In case of longitudinal waves, it is the distance between two consecutive condensations or rarefactions.

** In case of transverse waves, it is the distance between two consecutive crests or troughs.

e) **Velocity** (v) of a wave is the distance traveled by the wave in one second.

Note: 1) Velocity, $v = \nu\lambda$

2) *When a wave passes from one medium to another, the velocity v and wavelength λ changes but frequency ν remains the same.*

Speed of wave motion.

1. Speed of transverse wave in a stretched string is given by $v = \sqrt{\frac{T}{m}}$; where T = tension in the string and m = linear density or mass per unit length of the string.

The linear density m is relates to volume density ρ as $m = a\rho$; a = area of cross section of string.

2. Velocity of longitudinal wave (Sound wave) in a medium, $v = \sqrt{\frac{E}{\rho}}$; where E = modulus of elasticity and ρ = density of medium.

(a) In solids, $v = \sqrt{\frac{Y}{\rho}}$; Y = Young's modulus.

(b) In liquids, $v = \sqrt{\frac{B}{\rho}}$; B = Bulk modulus.

(c) In gas: According to Newton, velocity of sound waves, $v = \sqrt{\frac{P}{\rho}}$; P = Pressure, ρ = density.

Substuting values of P and ρ, velocity of sound in air = 280 m/s at STP. The experimental value of velocity of sound in air at STP is about 332 m/s. ie result obtained by Newton's formula is 16% lower than experimental value.

Laplace Correction:

Laplace modified Newton's formula taking into account the adiabatic change.

According to Laplace, $v = \sqrt{\frac{\gamma P}{\rho}}$; where $\gamma = \frac{C_p}{C_v}$

This equation is called Newton – Laplace equation.

For air, $\gamma = 1.4$. Hence velocity of sound at STP = 331.5 m/s. This agrees with the experimental result.

**** Equation of wave motion**.

In case of wave motion, particles of medium execute SHM. *A relation between the instantaneous displacement of a particle executing SHM and time is called equation of wave*.

The equations of wave motion in different forms are given by:

a) $y = A\sin(\omega t \pm \phi)$; where ϕ is called 'phase constant'.

b) $y = A\sin\left(\omega t \pm \frac{2\pi x}{\lambda}\right)$

ie $y = A\sin(\omega t \pm kx)$; where $k = \frac{2\pi}{\lambda}$

c) $y = A\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$

d) $y = A\sin\frac{2\pi}{\lambda}(vt \pm x)$

Note: **Path difference**

λ cm = phase diff of 2π radian.

1 cm = phase diff of $\frac{2\pi}{\lambda}$ radian.

X cm = phase diff of $\frac{2\pi X}{\lambda}$ radian.

NB **Principle of superposition.

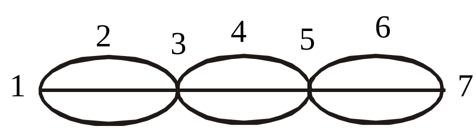
The phenomenon of intermixing of two or more waves to produce a new wave is called superposition of waves. If $y_1, y_2, y_3, \ldots\ldots$ are the displacements due to individual waves, the resultant displacement is given by $Y = y_1 + y_2 + y_3 + \ldots\ldots$ Hence the resultant displacement is the algebraic sum of the displacements of individual waves. This principle is called superposition principle.

The three important applications of superposition of waves are (1) Interference. (2) Stationary waves (3) Beats.

NB **Stationary waves OR Standing waves.

When two waves of the same frequency and amplitude travel in the opposite directions in a straight line at the same speed, their superposition gives rise to a new type of wave called stationary wave or standing wave.

In practice, stationary waves are set up in a medium when a progressive wave and its reflected wave get superimposed. These waves do not advance through the medium. Hence they are called stationary waves. Figure shows a standing wave:



The position of the particles 1,3,5, 7 etc which always remain at their mean positions are called nodes. Node is a position of zero displacement and maximum strain.

The positions of the particles 2,4,6 etc, which vibrate simple harmonically with maximum amplitude, are called antinodes. At antinodes, strain is maximum.

Distance between any two consecutive nodes or antinodes is equal to $\frac{\lambda}{2}$. Between a node and an antinode, the amplitude gradually increases from zero to maximum.

NB **Stationary waves in gas columns.

Closed tubes:

A tube closed at one end and open at the other end is called a closed tube or closed pipe. Consider a closed tube of length L containing a gas. Let the closed end correspond to x = 0, and the open end to x = L. Let a disturbance be produced at the open end.

The wave advancing though the gas column contained in the tube gets reflected at the closed end. The incident and reflected waves get superposed and stationary waves are formed. Always a node if formed at the closed end and an antinode at the open end.

Let the incident wave be represented by $y_1 = A \sin \frac{2\pi}{\lambda}(vt - x)$

Then the reflected wave is given by $y_2 = -A \sin \frac{2\pi}{\lambda}(vt + x)$.

The resultant wave is given by $y = y_1 + y_2 = -2A \sin\left(\frac{2\pi x}{\lambda}\right) \cos\left(\frac{2\pi v t}{\lambda}\right)$ ....(1)

At x = 0, y = 0 ie closed end corresponds to a node. Similarly, an antinode must be formed at x = L.

At antinode, the displacement y must be maximum. For this, amplitude $2A \sin\left(\frac{2\pi x}{\lambda}\right)$ must be maximum

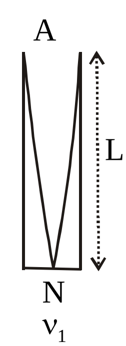
or $\sin\left(\frac{2\pi x}{\lambda}\right)$ must be maximum.

For maximum, $\sin\left(\frac{2\pi x}{\lambda}\right) = 1$

At open end, x = L $\sin\left(\frac{2\pi L}{\lambda}\right) = 1$

ie $\frac{2\pi L}{\lambda} = (2n+1)\frac{\pi}{2}$; where n = 0,1,2,3,.......

$\therefore \lambda_n = \frac{2\pi L}{(2n+1)} \frac{2}{\pi} = \frac{4L}{(2n+1)}$ ....(2) where n = 0,1,2,3,.......



Case (1): For first mode of vibration, n = 0.

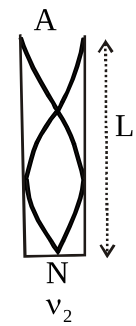
$\therefore$ from (2) $\lambda_1 = 4L$. So frequency, $\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}}$

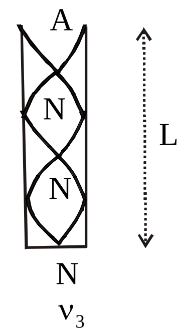
This frequency is called **fundamental frequency or the first harmonic.** *It is the smallest frequency with which the gas column in a closed pipe can vibrate*.

The formation of stationary waves when the air in closed tube vibrates in the first mode is as shown in fig.

Case (2) : For second mode of vibration, n = 1.

$\therefore \lambda_2 = \frac{4L}{3}$ Frequency $\nu_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\nu_1$.





This frequency is called **third harmonic or first overtone.**

Similarly for third mode of vibration, n =2.

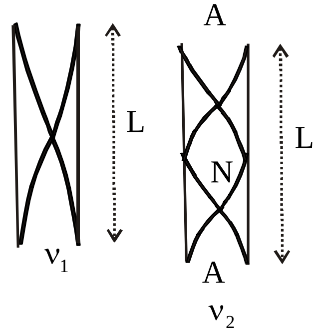
$\therefore \lambda_3 = \frac{4L}{5}$ and $\nu_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5\nu_1$

This is called **fifth harmonic or second overtone** and so on.

Thus for a closed pipe, only odd harmonics are present.

(2) Open pipe.

In open pipe, antinodes are formed at both ends of the pipe. The first wave pattern that can be formed is as follows. This occurs when $L = \frac{\lambda_1}{2}$ Or $\lambda_1 = 2L$



$\therefore \quad \nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ This is the fundamental frequency of first harmonic.

For the second harmonic, $L = \lambda_2 \quad \therefore \quad \nu_2 = \frac{v}{\lambda_2} = \frac{2v}{2L} = \frac{v}{L}$

For the third harmonic, the frequency is $\nu_3 = 3\frac{v}{2L}$ and so on. Thus the frequencies that are possible in an open pipe are in the ratio 1 : 2 : 3 : 4 : .... of the fundamental frequency $\frac{v}{2L}$.

Thus in case of an open pipe, all harmonics are possible.

Stationary waves in strings.

Consider a string stretched between two fixed points. When the string is plucked, a transverse progressive wave is produced in the string. This wave travels along the string and gets reflected at the fixed ends. The incident and the reflected waves superpose and stationary wave is formed.

The equation of stationary wave is $y = -2A\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$

From this equation, y = 0 if $\sin\frac{2\pi x}{\lambda} = 0$.

But we know sin nπ = 0 where n = 0,1,2,3.......

$\therefore \quad y = 0 \text{ if } \frac{2\pi x}{\lambda} = n\pi \quad \text{or} \quad x = \frac{n\lambda}{2}$

Thus at x = 0, $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$............... the displacement of the particle is zero. Such points are called as **nodes**.

y becomes maximum when $\sin\frac{2\pi x}{\lambda} = 1$, which happens when $\frac{2\pi x}{\lambda} = (2n+1)\frac{\pi}{2}$ or $x = \frac{(2n+1)\lambda}{4}$

Thus at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$,.............. the displacement is maximum. Such points are called **antinodes**.

For a stretched string of length L fixed at both ends, the two ends of the string have to be nodes. Thus the string can vibrate only some special patterns known as normal modes.

If the ends are chosen at x = 0 and x = L, then the wavelength λ must satisfy the condition $L = \frac{n\lambda}{2}$; where n = 1,2,3.............

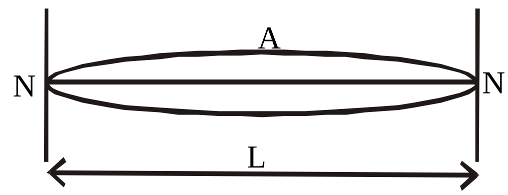
Thus $\lambda = \frac{2L}{n}$ for n = 1,2,3.................

The frequencies corresponding to these wave lengths are $\nu = \frac{v}{\lambda} = \frac{v}{2L/n}$ i.e. $\nu = \frac{nv}{2L}$

Thus the string can vibrate with only these frequencies.

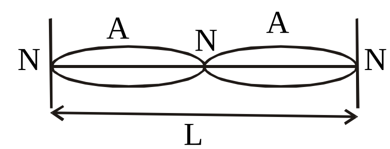
Special Cases

(i) When n = 1, $\nu_1 = \dfrac{v}{2L}$ This occurs when $\lambda_1 = 2L$.



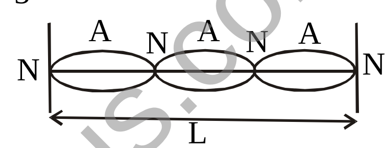
It is called the **fundamental frequency or first harmonic**.

(ii) when n = 2, $\nu_2 = \dfrac{2v}{2L} = \dfrac{v}{L}$ i.e. $\nu_2 = 2\nu_1$.



This occurs when $\lambda_2 = L$. This frequency is called the **second harmonic or first overtone**.

(iii) when n = 3, $\nu_3 = \dfrac{3v}{2L}$ i.e. $\nu_3 = 3\nu_1$. This occurs when $\lambda_3 = \dfrac{2L}{3}$.



This frequency is called the **third harmonic or second overtone**.
Thus in case of a stretched string, **all harmonics are possible.**

NB **Beats

When two sound waves of nearly equal frequencies traveling in a medium along the same direction superimpose, the intensity of the resultant sound at a particular position rises and falls regularly with time.

The phenomenon of regular variation in the intensity of sound with time at a particular position when two sound waves of nearly equal frequencies superimpose each other is called beats.

The number of beats produced per second is called **beat frequency**. It is seen that beat frequency is equal to the difference in the frequencies of the two waves. If ν_1 and ν_2 are the individual frequencies, then beat frequency = $\nu_1 - \nu_2$. When beat frequency is greater than 10, beats cannot be detected by human ear. Musicians use the beat phenomenon in tuning their instruments.

NB **Doppler effect in sound.

Let a source of sound produce a note of frequency ν. It produces ν waves per second in the medium. A listener at a distance from the source will receive ν waves per second, provided the source of sound, the listener and the medium through which sound travels are all at rest ie the frequency of sound received by the listener is also ν.

But if the source, listener or the medium is in motion, the listener feels an apparent change in the frequency of sound. When the source approaches the listener or vice versa, when both listener and source approach each other, the pitch of sound as heard by the listener is higher than the actual pitch of the sound. Similarly, when source moves away from listener or vice versa, the apparent pitch is lower than the actual pitch.

The apparent change in the pitch or the frequency of sound produced by a source due to relative motion of the source, listener or the medium is called Doppler effect.

The expression for apparent frequency, $\nu^1 = \nu\left(\dfrac{v-\ell}{v-S}\right)$; where, ν = actual frequency of source, v = velocity of sound, ℓ = velocity of listener, S = velocity of source.

Note: *If source is moving towards listener, its velocity, S is + ve. If source moves away, S is -ve. If listener moves towards source, its velocity, ℓ is -ve and if listener moves away from source, ℓ is + ve.*

Various cases:..............

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