

JAIPUR EDUCATION PLUS

Not Just Education but Education Plus....

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UNIT AND MEASUREMENT

- The dimensions of a physical quantity are the powers to which the fundamental (base) quantities are raised to represent that quantity.
- **Dimensionless Physical Quantities** : Angle, solid angle, relative density, specific gravity, strain, Poisson's ratio, Reynold's number, all trigonometric ratios, refractive index, mechanical efficiency, relative permittivity, dielectric constant, relative permeability, electric susceptibility, magnetic susceptibility.
- The **three main uses** of dimensional analysis are :

- (i) **Conversion of one system of units into another for which we use**

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

where M_1, L_1, T_1 are fundamental units on one system; M_2, L_2, T_2 are fundamental units on the other system; a, b, c are the dimensions of the quantity in mass, length and time; n_1 is numerical value of the quantity in one system and n_2 is its numerical value in the other system.

- (ii) Checking the dimensional correctness of a given physical relation.
 - (iii) Derivation of formulae.
- **Principle of Homogeneity of Dimensions** : According to this principle, a correct dimensional equation must be homogeneous, i.e., dimensions of all the terms in a physical expression must be same

$$\text{LHS} = \text{RHS}$$

- **Significant Figures** : In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which is doubtful, are called the significant figures. For counting significant figures rules are as :

1. All the non-zero digits are significant. In 2.738 the number of significant figures is 4.
 2. All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. As examples 209 and 3.002 have 3 and 4 significant figures respectively.
 3. If the measurement number is less than 1, the zero(s) on the right of decimal point and to the left of the first non-zero digit are non-significant. In 0.00807, first three underlined zeros are non-significant and the number of significant figures is only 3.
 4. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 12.3 m = 1230 cm = 12300 mm has only 3 significant figures.
 5. The trailing zero(s) in number with a decimal point are significant. Thus, 3.800 kg has 4 significant figures.
 6. A choice of change of units does not change the number of significant digits or figures in a measurement.
- In any mathematical operation involving addition, subtraction, decimal places in the result will correspond to lowest number of decimal places in any of the numbers involved.
 - In a mathematical operation like multiplication and division, number of significant figures in the product or in the quotient will correspond to the smallest number of significant figures in any of the numbers involved.
 - Problems with *accuracy* are due to errors. The *precision* describes the limitation of the measuring instrument.
 - Difference between measured value and true value of a quantity represents error of measurement.

(i) Mean of n measurements

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$= \frac{1}{n} \sum_{i=1}^n a_i$$

(ii) Mean absolute error

$$[|\Delta a_1| + |\Delta a_2| + |\Delta a_3|$$

$$\Delta a_{\text{mean}} = \frac{[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|]}{n}$$

$$= \frac{\sum_{l=1}^n |\Delta a_l|}{n}$$

(iii) Relative error = $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$ (iv) Percentage error = $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

- The errors are communicated in different mathematical operations as detailed below :

if $\pm \Delta a$, $\pm \Delta b$ and $\pm \Delta x$ are absolute errors in a , b and x respectively, then

$$(i) \text{ for } x = (a + b), \quad \Delta x = \pm [\Delta a + \Delta b]$$

$$(ii) \text{ for } x = (a - b), \quad \Delta x = \pm [\Delta a + \Delta b]$$

$$(iii) \text{ for } x = a \times b, \quad \frac{\Delta x}{x} = \pm \left[\frac{\Delta a}{a} + \frac{\Delta b}{b} \right]$$

$$(iv) \text{ for } x = a/b, \quad \frac{\Delta x}{x} = \pm \left[\frac{\Delta a}{a} + \frac{\Delta b}{b} \right]$$

$$(v) \text{ for } x = \frac{a^n b^m}{c^p}, \quad \frac{\Delta x}{x} = \pm \left[n \frac{\Delta a}{a} + m \frac{\Delta b}{b} + p \frac{\Delta c}{c} \right]$$

VERY SHORT ANSWER QUESTIONS (1 MARK)

- Name the strongest force in nature. What is its range?
- Give two discoveries of Physics used in your daily life.

3. What is the relation between light year and par sec.
 4. Give the order of magnitude of the following :
 - (i) size of atom
 - (ii) size of our galaxy.
 5. How many kg make 1 unified atomic mass unit?
 6. Name same physical quantities that have same dimension.
 7. Name the physical quantities that have dimensional formula $[ML^{-1} T^{-2}]$
 8. Give two examples of dimension less variables.
 9. State the number of significant figures in
 - (i) $0.007m^2$
 - (ii) $2.64 \times 10^{24} \text{ kg}$
 - (iii) 0.2370 g cm^{-3}
 - (iv) $0.2300m$
 - (v) 86400
 - (vi) 86400 m
 10. Given relative error in the measurement of length is .02, what is the percentage error?
 11. If a physical quantity is represented by $X = M^a L^b T^{-c}$ and the percentage errors in the measurements of Mm L and T are α , β and γ . What will be the percentage error in X.
 12. A boy recalls the relation for relativistic mass^(m) in terms of rest mass (m_0), velocity of particle v , but forgets to put the constant c (velocity of light). He writes $m = \frac{m_0}{(1 - v^2)^{1/2}}$ correct the equation by putting the missing 'c'.
 13. Name the technique used in locating.
 - (a) an under water obstacle
 - (b) position of an aeroplane in space.
 14. Deduce dimensional formulae of—
 - (i) Boltzmann's constant
 - (ii) mechanical equivalent of heat.
 15. Give examples of dimensional constants and dimensionless constants.
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SHORT ANSWER QUESTIONS (2 MARKS)

1. What is a physical standard? What characteristics should it have?
2. Define the term unit. Distinguish between fundamental and derived units.
3. Describe the principle and use of SONAR and RADAR.
4. State the principle of homogeneity. Test the dimensional homogeneity of equations—

$$(i) \quad S = ut + \frac{1}{2}at^2$$

$$(ii) \quad S_n = u + \frac{a}{2}(2n - 1)$$

5. In van der Waals's gas equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. Determine the dimensions of a and b .
6. Using dimensions convert (a) 1 newton into dynes (b) 1 erg into joules.
7. Magnitude of force experienced by an object moving with speed v is given by $F = kv^2$. Find dimensions of k .
8. A book with printing error contains four different formulae for displacement. Choose the correct formula/formulae

$$(a) \quad y = a \sin \frac{2\pi}{T} t$$

$$(b) \quad y = a \sin vt$$

$$(c) \quad y = \frac{a}{T} \sin \left(\frac{t}{a}\right)$$

$$(d) \quad y = \frac{a}{T} \left(\sin \frac{2\pi}{T} t + \cos \frac{2\pi}{T} t \right)$$

9. Give limitations of dimensional analysis.
10. For determination of ' g ' using simple pendulum, measurements of length and time period are required. Error in the measurement of which quantity will have larger effect on the value of ' g ' thus obtained. What is done to minimise this error?

SHORT ANSWER QUESTIONS (3 MARKS)

1. Give the name of six Indian Scientists and their discoveries.
2. Name the discoveries made by the following scientists :
 - (a) Faraday
 - (b) Chadwick
 - (c) Hubble
 - (d) Maxwell
 - (e) Newton
 - (f) Bohr.
3. Name the scientific principle on which the following technology is based.
 - (i) Steam engine
 - (ii) Laser
 - (iii) Aeroplane
 - (iv) Rocket propulsion
 - (v) Radio and T.V.
 - (vi) Production of Ultra high magnetic field.
4. Describe a method for measuring the molecular size of Oleic acid.
5. What types of phenomena can be used as a time standard. What are the advantages of defining second in terms of period of radiation from cesium -133 atom.
6. Deduce the dimensional formula for the following quantities
 - (i) Gravitational constant
 - (ii) Young's modulus
 - (iii) Coefficient of viscosity.
7. Define the following units :
 - (i) Light year
 - (ii) Parsec
 - (iii) Astronomical unit (Au)

LONG ANSWER QUESTIONS (5 MARKS)

1. Name the four basic forces in nature. Write a brief note of each. Hence compare their strengths and ranges.
 2. Distinguish between the terms precision and accuracy of a measurement.
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3. Explain

- (i) absolute error
- (ii) mean absolute error
- (iii) relative error
- (iv) percentage error
- (v) random error

NUMERICALS

1. Determine the number of light years in one metre.
2. The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.
3. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces 20.15 g and 20.17 g are added to the box.
 - (i) What is the total mass of the box?
 - (ii) The difference in masses of the pieces to correct significant figures.
4. 5.74 g of a substance occupies 1.2 cm^3 . Express its density to correct significant figures.
5. If displacement of a body $s = (200 \pm 5) \text{ m}$ and time taken by it $t = (20 + 0.2) \text{ s}$, then find the percentage error in the calculation of velocity.
6. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy.
7. The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m and 2.48m. Find the average length, absolute error and percentage error. Express the result with error limit.
8. A physical quantity is measured as $Q = (2.1 \pm 0.5)$ units. Calculate the percentage error in (i) Q^2 (2) $2Q$.
9. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be $35.72''$ of arc. Calculate diameter of Jupiter.
10. A laser light beamed at the moon takes 2.56 s to return after reflection at the moon's surface. What will be the radius of lunar orbit.

11. Convert

(i) $3\text{m} \cdot \text{S}^{-2}$ to km h^{-2}

(ii) $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ to $\text{dyne cm}^2 \text{ g}^{-2}$

12. A calorie is a unit of heat or energy and it equals 4.2 J where $1\text{J} = 1\text{kg m}^2\text{S}^{-2}$. Suppose we employ a system of units in which unit of mass is α kg, unit of length is β m, unit of time is γ s. What will be magnitude of calorie in terms of this new system.

13. The escape velocity v of a body depends on—

(i) the acceleration due to gravity 'g' of the planet,

(ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.

14. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

15. One mole of an ideal gas at STP occupies 22.4 L. What is the ratio of molar volume to atomic volume of a mole of hydrogen? Why is the ratio so large. Take radius of hydrogen molecule to be 1\AA .

VERY SHORT ANSWER (1 MARK)

3. 1 parsec = 3.26 light year

4. Size of atom = 10^{-10} m (b) size of galaxy = 10^{22} m

Order = -10, Order of magnitude = 22.

5. $1u = 1.66 \times 10^{-27}$ kg

6. Work, energy and torque.

7. Stress, pressure, modulus of elasticity.

8. Strain, refractive index.

9. (i) 1, (ii) 3, (iii) 4, (iv) 4, (v) 3, (vi) 5 since it comes from a measurement the last two zeros become significant.

10. 2%

11. % error in measurement of $X = a\alpha + b\beta + c\gamma$.
12. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from 1 but v^2/c^2 can be subtracted from 1.

$$\therefore m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

13. (a) SONAR \rightarrow Sound Navigation and Ranging.
 (b) RADAR \rightarrow Radio Detection and Ranging.
14. (i) Boltzmann Constant :

$$k = \frac{\text{Heat}}{\text{Temperature}} \Rightarrow [k] = \frac{ML^2T^{-2}}{K} = M^1L^2T^{-2}K^{-1}$$

$$(ii) [J] = \left[\frac{\text{Work}}{\text{Heat}} \right] = \frac{M^1L^2T^{-2}}{M^1L^2T^{-2}} = [m^0L^0T^0]$$

15. Dimensional Constants : Gravitational constant, plank's constant.
 Dimensionless Constants : π , e .

SHORT ANSWER (2 MARKS)

4. (i) Dimension of L.H.S. = [s] = $[M^0L^1T^0]$
 Dimension of R.H.S = [ut] + [at²] = $[M^0L^1T^{-1}] + [M^0L^1T^{-2} \cdot T^2]$
 $= M^0L^1T^0$

\therefore The equation to dimensionally homogeneous.

- (ii) S_n = Distance travelled in n^{th} sec that is $(S_n - S_{n-1})$

$$\therefore S_n = u \times 1 + \frac{a}{2} (2n - 1) \times 1$$

Hence this is dimensionally incorrect.

5. Since dimensionally similar quantities can only be added

$$\therefore [P] = \left[\frac{a}{v^2} \right] \Rightarrow [a] = [Pv^2] = M^1 L^5 T^{-2}$$

$$[b] = [v] = L^3$$

$$7. [k] = \frac{[F]}{[v^2]} = \frac{M^1 L^1 T^{-2}}{[L T^{-1}]^2} = \frac{M^1 L^1 T^{-2}}{M^0 L^2 T^{-2}} = [M^1 L^{-1}]$$

8. The argument of sine and cosine function must be dimensionless so (a) is the probable correct formula. Since

$$(a) y = a \sin\left(\frac{2\pi}{T} t\right) \quad \therefore \left[\frac{2\pi t}{T}\right] = [T^0] \text{ is dimensionless.}$$

$$(b) y = a \sin \mu t \quad \therefore [\mu t] = [L] \text{ is dimensional so this equation is incorrect.}$$

$$(c) y = \frac{a}{t} \sin\left(\frac{t}{a}\right) \quad \left[\frac{t}{a}\right] \text{ is dimensional so this is incorrect.}$$

$$(d) y = \frac{a}{t} \left(\sin \frac{2\pi}{T} t + \cos \frac{2\pi t}{T} \right) : \text{ Though } \frac{2\pi t}{T} \text{ is dimensionless } \frac{a}{t} \text{ does not have dimensions of displacement so this is also incorrect}$$

9. Limitation of dimensional analysis :-

1. The value of proportionality constant cannot be obtained
2. Equation containing sine and cosine, exponents, logx etc cannot be analysed.
3. It fails to derive the exact form of physical relation which depends on more than three fundamental quantities
4. It does not tell whether a quantity is scalar or vector.

ANSWERS FOR NUMERICALS

(NUMERICAL)

1. $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

$$1 \text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16} \text{ ly}$$

$$\begin{aligned}
 2. \quad P &= 2(l + b) \\
 &= 2(10.5 + 5.2) + 2(0.2 + 0.1) \\
 &= (31.4 + 0.6) \text{ cm}
 \end{aligned}$$

3. (i) Mass of box : 2.3 kg

$$\text{Mass of gold pieces} = 20.15 + 20.17 = 40.32 \text{ g} = 0.04032 \text{ kg.}$$

$$\text{Total mass} = 2.3 + 0.04032 = 2.34032 \text{ kg}$$

In correct significant figure mass = 2.3 kg (as least decimal)

(ii) Difference in mass of gold pieces = 0.02 g

In correct significant figure (2 significant fig. minimum decimal) will be 0.02 g.

$$4. \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{5.74}{1.2} = 4.783 \text{ g/cm}^3$$

Here least significant figure is 2, so density = 4.8 g/cm³

$$5. \quad \text{Percentage error in measurement of displacement} = \frac{5}{200} \times 100$$

$$\text{Percentage error in measurement of time} = \frac{0.2}{20} \times 100$$

$$\therefore \text{Maximum permissible error} = 2.5 + 1 = 3.5\%$$

$$6. \quad \text{K.E.} = \frac{1}{2} mv^2$$

$$\therefore \frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} \Rightarrow \frac{\Delta k}{k} \times 100 = \frac{\Delta m}{m} \times 100 + 2 \left(\frac{\Delta v}{v} \right) \times 100$$

$$\therefore \text{Percentage error in K.E.} = 3\% + 2 \times 2\% = 7\%$$

7. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} = 2.48 \text{ m}$$

Mean absolute error

$$\frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.013n$$

$$\begin{aligned}\text{Percentage error} &= \frac{0.01}{2.48} \times 100\% = 0.04 \times 100\% \\ &= 0.40\%\end{aligned}$$

$$\text{Correct length} = (2.48 \pm 0.01)\text{m}$$

$$\text{Correct length} = (2.48 \text{ M} \pm 0.40\%)$$

8. $P = Q^2$

$$\frac{\Delta p}{p} = \frac{2\Delta Q}{Q} \quad \left(\frac{0.5}{2.1}\right) = \frac{1.0}{2.1} = 0.476$$

$$\frac{\Delta p}{p} \times 100\% = 47.6\% = 48\%$$

$$R = 2Q$$

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \Rightarrow \frac{0.5}{2.1} = 0.238$$

$$\frac{\Delta R}{R} \times 100\% = 24\%$$

9. $\theta = 35.72''$

$$1'' = 4.85 \times 10^{-6} \text{ radian} \Rightarrow = 35.72 \times 4.85 \times 10^{-6}$$

$$\begin{aligned}d = D\theta &= 824.7 \times 10^6 \times 35.72 \times 4.85 \times 10^{-6} \\ &= 1.4287 \times 10^5 \text{ km}\end{aligned}$$

10. $t = 2.56 \text{ s}$

$$t' = \text{time taken by laser beam to go to the man} = \frac{t}{2}$$

$$\text{distance between earth and moon} = d = c \times \frac{t}{2}$$

$$= 3 \times 10^8 \times \frac{2.56}{2}$$

$$= 3.84 \times 10^8 \text{ m}$$

$$11. \quad (i) \quad 3 \text{ m s}^{-2} = \left(\frac{3}{1000} \text{ km} \right) \left(\frac{1}{60 \times 60} \text{ hr} \right)^{-2}$$

$$= \frac{3 \times (60 \times 60)^2}{1000} = 3.9 \times 10^4 \text{ km h}^{-2}$$

$$(ii) \quad G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$= 6.67 \times 10^{-11} (\text{kg m s}^{-2})(\text{m}^2 \text{ kg}^{-2})$$

$$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$

$$= 6.67 \times 10^{-11} (1000 \text{ g})^{-1} (100 \text{ cm})^3 (\text{s}^{-2})$$

$$= 6.67 \times 10^{-11} \times \frac{1}{1000} \times 100 \times 100 \times 100$$

$$= 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}$$

$$12. \quad n_2 = n_1 \left[\frac{m_1}{m_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$= 4.2 \left(\frac{\text{kg}}{\alpha \text{ kg}} \right)^1 \left(\frac{\text{m}}{\beta \text{ m}} \right)^2 \left(\frac{\text{s}}{\gamma \text{ s}} \right)^{-2}$$

$$n_2 = 4.2 \alpha^{-1} \beta^{-2} \gamma^{+2}$$

$$13. \quad v \propto g^a R^b \quad \mu = k g^a R^b \quad K \rightarrow \text{dimensionless proportionality constant}$$

$$[v] = [g]^a [R]^b$$

$$[M^0 L^1 T^{-1}] = [M^0 L^1 T^{-2}]^a [m^0 L^1 T^0]^b$$

equating powers

$$1 = a + b$$

$$-1 = -2a \Rightarrow a = \frac{1}{2}$$

$$b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore v = k\sqrt{gR}$$

$$14. n \propto l^a T^b m^c [l] = M^0 L^1 T^0$$

$$[T] = M^1 L^1 T^{-2} \text{ (force)}$$

$$[M] = M^1 L^{-1} T^0$$

$$[M^0 L^1 T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$$

$$b + c = 0$$

$$a + b - c = 0$$

$$-2b = -1 \quad b = 1/2$$

$$c = -\frac{1}{2} \quad a = 1$$

$$n \propto \frac{1}{l} \sqrt{\frac{T}{M}}$$

$$15. 1 \text{ A}^0 = 10^{-10} \text{ m}$$

Atomic volume of 1 mole of hydrogen = Avagadios number \times volume of hydrogen molecule

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times \pi \times (10^{-10})^3$$

$$= 25.2 \times 10^{-7} \text{ m}^3$$

$$\text{Molar volume} = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} = 0.89 \times 10^4 \approx 10^4$$

This ratio is large because actual size of gas molecule is negligible in comparison to the inter molecular separation.