

CBSE Sample Paper-03

Time allowed: 3 hours

Mathematics

Maximum Marks: 100

Class – XII

General Instructions:

- All questions are compulsory.
- The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- Use of calculators is not permitted.

Section A

- Is R defined on the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y): y \text{ is divisible by } x\}$ symmetric.
- Calculate the direction cosines of the vector $\vec{a} = 3i - 2j - 5k$.
- What is the principal value branch of $\cos^{-1} x$?
- Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.
- Evaluate without expanding $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
- Find $A' = [-2 \ 4 \ 5]$, $B' = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ find $(AB)'$.

Section B

- Using properties of determinants prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = abc + bc + ca + ab$$

8. Find the equations of all lines having slope 0 and that are tangent to the curve $y = \frac{1}{x^2 - 2x + 2}$.

9. If $f(x) = \frac{x-1}{x+1}$, ($x \neq 1, -1$), show that $f \circ f^{-1}$ is an identity function.

10. Find $\frac{dy}{dx}$ if $\log(xy) = x^2 + y^2$.

11. Prove that $\tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$, $|x| < \frac{1}{\sqrt{3}}$

12. If A, B, C have the co-ordinates (2,0,0) , (0,1,0) , (0,0,2) , then show that ABC is an isosceles triangle.

13. (a) If A and B are two events defined on a sample space s.t.

$$P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}, P(B^c) = \frac{1}{3}, \text{ find } P(A).$$

(b) If A and B are two events defined on a sample space s.t.

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A \text{ and } B) = \frac{1}{8}, \text{ find } P(\text{not } A \text{ and not } B).$$

14. Find all intervals on which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is (a) strictly increasing (b) strictly decreasing.

15. Solve the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

16. Show that $(|\vec{a}| \vec{b} + |\vec{b}| \vec{a}) \cdot (|\vec{a}| \vec{b} - |\vec{b}| \vec{a}) = 0$

17. Integrate $\int \log(1+x^2) dx$.

18. Find the shortest distance between the lines l_1 and l_2 given by :

$$\vec{r} = (i + 2j + k) + \lambda(i - j + k)$$

$$\vec{r} = (2i - j - k) + \mu(2i + j + 2k)$$

19. Find the vector equation of the plane passing through the points $i+j-k$ and $2i+6j+k$ and parallel to the line $\vec{r} = (3i - 5j + k) + \lambda(i - 2j + k)$

Section C

20. A factory can hire two tailors A and B in order to stitch pants and shirts. Tailor A can stitch 6 shirts and 4 pants in a day. Tailor B can stitch 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.
21. Find the area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$
22. A, Band C play a game and the chances of winning in it in an attempt are $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$ respectively. A has the first chance, followed by Band C. The cycle is repeated till one of them wins the game. Find their respective chances of winning the game.
23. If $x = a \sec^3 \theta$, $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.
24. Integrate $\int \frac{x^2 dx}{(x+3)\sqrt{3x+4}}$
25. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
26. Solve the following system of equations using matrix method
- $$\begin{aligned}x + y + z &= 4 \\2x - y + z &= -1 \\2x + y - 3z &= -9\end{aligned}$$