

**CBSE GUIDE Sample  
Paper-05 Mathematics**

Time allowed: 3 hours

Maximum Marks: 100

**General Instructions:**

**Class – XII**

- All questions are compulsory.
- The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- Use of calculators is not permitted.

**Section A**

- Is R defined on the set  $A=\{1,2,3,\dots,14,15\}$  defined as  $R=\{(x,y):3x-y=0\}$  reflexive?
- Find the angle between the vectors  $\vec{a} = 4i + 4j$  and  $\vec{b} = 4i - 2j$ .
- Evaluate  $\sin^{-1}\left(\frac{1}{6}\right) + \cos^{-1}\left(\frac{1}{6}\right)$ ?
- If a matrix has 12 elements, what are the possible orders it can have?
- Prove that  $A-A'$  is skew symmetric.
- Find the values of x,y,z s.t  $\begin{bmatrix} x-y & 0 & 0 \\ z & 6 & 0 \\ 0 & 0 & 2y \end{bmatrix}$  is a scalar matrix?

**Section B**

- Solve:  $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$
- Find a unit vector perpendicular to each of the vectors  $(\vec{a} - \vec{b})$  and  $(\vec{a} + \vec{b})$  where  $\vec{a} = i + j + k$  and  $\vec{b} = i + 2j + 3k$ .
- If A and B are independent events such that  $P(A \cup B) = 0.6$ ,  $P(A) = 0.2$ . Find  $P(B)$

10. The relation between the total cost  $y$  and the total output  $x$  is given by  

$$y = \frac{3x(x+7)}{x+5} + 5.$$
Prove that the marginal cost continuously falls as output increases.
11. Solve  $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$
12. If  $A = \begin{bmatrix} 0 & -\tan \alpha / 2 \\ \tan \alpha / 2 & 0 \end{bmatrix}$  and  $I$  is the identity matrix of order 2, show that  

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$
13. Find the equations of the tangent and the normal to the curve  $x = \cos t, y = \sin t$  at  $t = \frac{\pi}{4}$ .
14. Show that the relation  $R$  in the set  $\mathbb{Z}$  of integers given by  $R = \{(a, b) : 7 \text{ divides } a - b\}$  is an equivalence relation.
15. If  $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots}}}$  prove that  $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$ .
16. For any two vectors  $\vec{a}$  and  $\vec{b}$ , prove that  $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ .
17. Integrate  $\int \frac{e^x}{e^{2x} - 4} dx$ . Write ant points for promoting national integration.
18. Find the vector equation of the plane passing through the intersection of the planes  $\vec{r} \cdot (2\vec{i} + 2\vec{j} - 3\vec{k}) = 7, \vec{r} \cdot (2\vec{i} + 5\vec{j} + 3\vec{k}) = 9$  and the point  $(2, 1, 3)$ .
19. Find the equation of the plane passing through the line  $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$  and the point  $(0, 7, -7)$ .  
Show that the line  $x = \frac{7-y}{3} = \frac{z+7}{2}$  lies on the plane.

### Section C

20. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let  $\frac{3}{4}$  be the probability that he knows the answer and  $\frac{1}{4}$  be the probability that he guesses. Assume that a student who guesses the answer would answer correctly with probability  $\frac{1}{4}$ . What is the probability that a student knows the answer, given that he has answered it correctly.

21. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  w.r.t.  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

22. Integrate  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx$ .

23. Prove that the volume of the largest cone that can be inscribed in a sphere of radius  $a$  is  $\frac{8}{27}$  of the volume of the sphere.

24. Solve the following system of equations using matrix method

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

25. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and 2 unit per kg of vitamin C. Food 1 costs Rs.50 per kg and Food 2 costs Rs.70 per kg. Using linear programming , find the minimum cost of such a mixture.

26. Draw a rough sketch of the region  $\{(x, y) : y^2 < 4x, 4x^2 + 4y^2 \leq 9\}$  and find the area enclosed.