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CLASS 11th

CHAPTER

PHYSICS

KINETIC THEORY OF GASES

NCERT Solutions

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Exercises

13.1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3\AA .

Answer

Diameter of an oxygen molecule, $d = 3\text{\AA}$

Radius, $r = d/2 = 3/2 = 1.5\text{\AA} = 1.5 \times 10^{-8}\text{ cm}$

Actual volume occupied by 1 mole of oxygen gas at STP = 22400 cm^3

Molecular volume of oxygen gas, $V = (4/3)\pi r^3 N$

Where, N is Avogadro's number = 6.023×10^{23} molecules/mole

$\therefore V = (4/3) \times 3.14 \times (1.5 \times 10^{-8})^3 \times 6.023 \times 10^{23} = 8.51\text{ cm}^3$

Ratio of the molecular volume to the actual volume of oxygen = $8.51 / 22400$
 $= 3.8 \times 10^{-4}$.

13.2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP: 1 atmospheric pressure, $0\text{ }^\circ\text{C}$). Show that it is 22.4 litres.

Answer

The ideal gas equation relating pressure (P), volume (V), and absolute temperature (T) is given as:

$$PV = nRT$$

Where,

R is the universal gas constant = $8.314\text{ J mol}^{-1}\text{ K}^{-1}$

n = Number of moles = 1

T = Standard temperature = 273 K

P = Standard pressure = $1\text{ atm} = 1.013 \times 10^5\text{ Nm}^{-2}$

$$\therefore V = nRT / P$$

$$= 1 \times 8.314 \times 273 / (1.013 \times 10^5)$$

$$= 0.0224 \text{ m}^3$$

$$= 22.4 \text{ litres}$$

Hence, the molar volume of a gas at STP is 22.4 litres.

13.3. Figure 13.8 shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.

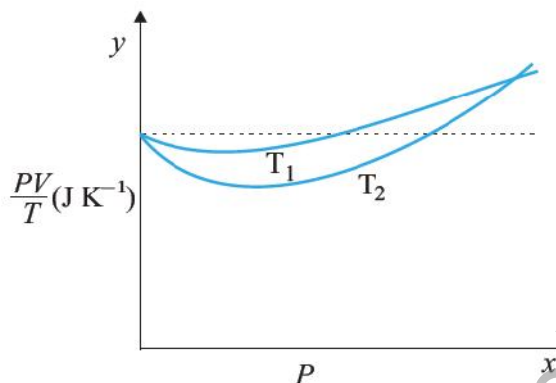


Fig. 13.8

- What does the dotted plot signify?
- Which is true: $T_1 > T_2$ or $T_1 < T_2$?
- What is the value of PV/T where the curves meet on the y -axis?
- If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y -axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)? (Molecular mass of $\text{H}_2 = 2.02\mu$, of $\text{O}_2 = 32.0\mu$, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

Answer

- The dotted plot in the graph signifies the ideal behaviour of the gas, i.e., the ratio PV/T is equal. μR (μ is the number of moles and R is the universal gas constant) is a constant quantity. It is not dependent on the pressure of the gas.
- The dotted plot in the given graph represents an ideal gas. The curve of the gas at temperature T_1 is closer to the dotted plot than the curve of the gas at temperature T_2 . A real gas approaches the behaviour of an ideal gas when its temperature increases. Therefore, $T_1 > T_2$ is true for the given plot.
- The value of the ratio PV/T , where the two curves meet, is μR . This is because the ideal gas equation is given as:

$$PV = \mu RT$$

$$PV/T = \mu R$$
 Where,
 P is the pressure
 T is the temperature

V is the volume

μ is the number of moles

R is the universal constant

Molecular mass of oxygen = 32.0 g

Mass of oxygen = 1×10^{-3} kg = 1 g

$R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$

$\therefore PV/T = (1/32) \times 8.314$

$= 0.26 \text{ J K}^{-1}$

Therefore, the value of the ratio PV/T , where the curves meet on the y -axis, is

0.26 J K^{-1} .

(d) If we obtain similar plots for 1.00×10^{-3} kg of hydrogen, then we will not get the same value of PV/T at the point where the curves meet the y -axis. This is because the molecular mass of hydrogen (2.02 u) is different from that of oxygen (32.0 u).

We have:

$PV/T = 0.26 \text{ J K}^{-1}$

$R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$

Molecular mass (M) of $\text{H}_2 = 2.02 \text{ u}$

$PV/T = \mu R$ at constant temperature

Where, $\mu = m/M$

$m =$ Mass of H_2

$\therefore m = (PV/T) \times (M/R)$

$= 0.26 \times 2.02 / 8.31$

$= 6.3 \times 10^{-2} \text{ g} = 6.3 \times 10^{-5} \text{ kg}$

Hence, $6.3 \times 10^{-5} \text{ kg}$ of H_2 will yield the same value of PV/T .

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13.4. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C . After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C . Estimate the mass of oxygen taken out of the cylinder ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, molecular mass of $\text{O}_2 = 32\mu$).

Answer

Volume of oxygen, $V_1 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$

Gauge pressure, $P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Pa}$

Temperature, $T_1 = 27^\circ\text{C} = 300 \text{ K}$

Universal gas constant, $R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$

Let the initial number of moles of oxygen gas in the cylinder be n_1 .

The gas equation is given as:

$P_1 V_1 = n_1 R T_1$

$\therefore n_1 = P_1 V_1 / R T_1$

$= (15.195 \times 10^5 \times 30 \times 10^{-3}) / (8.314 \times 300) = 18.276$

But $n_1 = m_1 / M$

Where,

m_1 = Initial mass of oxygen

M = Molecular mass of oxygen = 32 g

$$\therefore m_1 = n_1 M = 18.276 \times 32 = 584.84 \text{ g}$$

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces.

Volume, $V_2 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$

Gauge pressure, $P_2 = 11 \text{ atm} = 11 \times 1.013 \times 10^5 \text{ Pa}$

Temperature, $T_2 = 17^\circ\text{C} = 290 \text{ K}$

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as:

$$P_2 V_2 = n_2 R T_2$$

$$\therefore n_2 = P_2 V_2 / R T_2$$

$$= (11.143 \times 10^5 \times 30 \times 10^{-3}) / (8.314 \times 290) = 13.86$$

But $n_2 = m_2 / M$

Where,

m_2 is the mass of oxygen remaining in the cylinder

$$\therefore m_2 = n_2 M = 13.86 \times 32 = 453.1 \text{ g}$$

The mass of oxygen taken out of the cylinder is given by the relation:

Initial mass of oxygen in the cylinder – Final mass of oxygen in the cylinder

$$= m_1 - m_2$$

$$= 584.84 \text{ g} - 453.1 \text{ g}$$

$$= 131.74 \text{ g}$$

$$= 0.131 \text{ kg}$$

Therefore, 0.131 kg of oxygen is taken out of the cylinder.

13.5. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C . To what volume does it grow when it reaches the surface, which is at a temperature of 35°C ?

Answer

Volume of the air bubble, $V_1 = 1.0 \text{ cm}^3 = 1.0 \times 10^{-6} \text{ m}^3$

Bubble rises to height, $d = 40 \text{ m}$

Temperature at a depth of 40 m, $T_1 = 12^\circ\text{C} = 285 \text{ K}$

Temperature at the surface of the lake, $T_2 = 35^\circ\text{C} = 308 \text{ K}$

The pressure on the surface of the lake:

$$P_2 = 1 \text{ atm} = 1 \times 1.013 \times 10^5 \text{ Pa}$$

The pressure at the depth of 40 m:

$$P_1 = 1 \text{ atm} + d\rho g$$

Where,

ρ is the density of water = 10^3 kg/m^3

g is the acceleration due to gravity = 9.8 m/s^2

$$\therefore P_1 = 1.013 \times 10^5 + 40 \times 10^3 \times 9.8 = 493300 \text{ Pa}$$

We have $P_1V_1 / T_1 = P_2V_2 / T_2$

Where, V_2 is the volume of the air bubble when it reaches the surface

$$\begin{aligned} V_2 &= P_1V_1T_2 / T_1P_2 \\ &= 493300 \times 1 \times 10^{-6} \times 308 / (285 \times 1.013 \times 10^5) \\ &= 5.263 \times 10^{-6} \text{ m}^3 \text{ or } 5.263 \text{ cm}^3 \end{aligned}$$

Therefore, when the air bubble reaches the surface, its volume becomes 5.263 cm^3 .

13.6. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27°C and 1 atm pressure.

Answer

Volume of the room, $V = 25.0 \text{ m}^3$

Temperature of the room, $T = 27^\circ\text{C} = 300 \text{ K}$

Pressure in the room, $P = 1 \text{ atm} = 1 \times 1.013 \times 10^5 \text{ Pa}$

The ideal gas equation relating pressure (P), Volume (V), and absolute temperature (T) can be written as:

$$PV = k_BNT$$

Where,

k_B is Boltzmann constant = $1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$

N is the number of air molecules in the room

$$\begin{aligned} \therefore N &= PV / k_B T \\ &= 1.013 \times 10^5 \times 25 / (1.38 \times 10^{-23} \times 300) \\ &= 6.11 \times 10^{26} \text{ molecules} \end{aligned}$$

Therefore, the total number of air molecules in the given room is 6.11×10^{26} .

13.7. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million Kelvin (the typical core temperature in the case of a star).

Answer

(i) At room temperature, $T = 27^\circ\text{C} = 300 \text{ K}$

Average thermal energy = $(3/2)kT$

Where k is Boltzmann constant = $1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$

$$\begin{aligned} \therefore (3/2)kT &= (3/2) \times 1.38 \times 10^{-38} \times 300 \\ &= 6.21 \times 10^{-21} \text{ J} \end{aligned}$$

Hence, the average thermal energy of a helium atom at room temperature (27°C) is $6.21 \times 10^{-21} \text{ J}$.

(ii) On the surface of the sun, $T = 6000 \text{ K}$

Average thermal energy = $(3/2)kT$

$$\begin{aligned} &= (3/2) \times 1.38 \times 10^{-38} \times 6000 \\ &= 1.241 \times 10^{-19} \text{ J} \end{aligned}$$

Hence, the average thermal energy of a helium atom on the surface of the sun is $1.241 \times 10^{-19} \text{ J}$.

(iii) At temperature, $T = 10^7$ K

Average thermal energy = $(3/2)kT$

$$= (3/2) \times 1.38 \times 10^{-23} \times 10^7$$

$$= 2.07 \times 10^{-16} \text{ J}$$

Hence, the average thermal energy of a helium atom at the core of a star is 2.07×10^{-16} J.

13.8. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Answer

All the three vessels have the same capacity, they have the same volume.

Hence, each gas has the same pressure, volume, and temperature.

According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number, $N = 6.023 \times 10^{23}$.

The root mean square speed (v_{rms}) of a gas of mass m , and temperature T , is given by the relation:

$$v_{\text{rms}} = (3kT/m)^{1/2}$$

where,

k is Boltzmann constant

For the given gases, k and T are constants.

Hence v_{rms} depends only on the mass of the atoms, i.e.,

$$v_{\text{rms}} \propto (1/m)^{1/2}$$

Therefore, the root mean square speed of the molecules in the three cases is not the same. Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

13.9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20°C ? (atomic mass of Ar = 39.9 u, of He = 4.0 u).

Answer

Temperature of the helium atom, $T_{\text{He}} = -20^\circ\text{C} = 253$ K

Atomic mass of argon, $M_{\text{Ar}} = 39.9$ u

Atomic mass of helium, $M_{\text{He}} = 4.0$ u

Let, $(v_{\text{rms}})_{\text{Ar}}$ be the rms speed of argon.

Let $(v_{\text{rms}})_{\text{He}}$ be the rms speed of helium.

The rms speed of argon is given by:

$$(v_{rms})_{Ar} = \sqrt{\frac{3RT_{Ar}}{M_{Ar}}} \quad \dots(i)$$

where,

R is the universal gas constant

T_{Ar} is temperature of argon gas

The *rms* speed of helium is given by:

$$(v_{rms})_{He} = \sqrt{\frac{3RT_{He}}{M_{He}}} \quad \dots(ii)$$

It is given that:

$$(v_{rms})_{Ar} = (v_{rms})_{He}$$

$$\sqrt{\frac{3RT_{Ar}}{M_{Ar}}} = \sqrt{\frac{3RT_{He}}{M_{He}}}$$

$$\frac{T_{Ar}}{M_{Ar}} = \frac{T_{He}}{M_{He}}$$

$$T_{Ar} = \frac{T_{He}}{M_{He}} \times M_{Ar}$$

$$= \frac{253}{4} \times 39.9$$

$$= 2523.675 = 2.52 \times 10^3 \text{ K}$$

Therefore, the temperature of the argon atom is 2.52×10^3 K.

13.10. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of $N_2 = 28.0$ u).

Answer

$$\text{Mean free path} = 1.11 \times 10^{-7} \text{ m}$$

$$\text{Collision frequency} = 4.58 \times 10^9 \text{ s}^{-1}$$

$$\text{Successive collision time} \approx 500 \times (\text{Collision time})$$

$$\text{Pressure inside the cylinder containing nitrogen, } P = 2.0 \text{ atm} = 2.026 \times 10^5 \text{ Pa}$$

$$\text{Temperature inside the cylinder, } T = 17^\circ\text{C} = 290 \text{ K}$$

$$\text{Radius of a nitrogen molecule, } r = 1.0 \text{ \AA} = 1 \times 10^{10} \text{ m}$$

$$\text{Diameter, } d = 2 \times 1 \times 10^{10} = 2 \times 10^{10} \text{ m}$$

$$\text{Molecular mass of nitrogen, } M = 28.0 \text{ g} = 28 \times 10^{-3} \text{ kg}$$

The root mean square speed of nitrogen is given by the relation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where,

R is the universal gas constant = $8.314 \text{ J mole}^{-1} \text{ K}^{-1}$

$$\therefore v_{rms} = \frac{3 \times 8.314 \times 290}{28 \times 10^{-3}} = 508.26 \text{ m/s}$$

The mean free path (l) is given by relation:

$$l = \frac{KT}{\sqrt{2} \times d^2 \times P}$$

where,

k is the boltzmann constant = $1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$

$$\begin{aligned} \therefore l &= \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 2.026 \times 10^5} \\ &= 1.11 \times 10^{-7} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Collision frequency} &= v_{rms} / l \\ &= 508.26 / (1.11 \times 10^{-7}) = 4.58 \times 10^9 \text{ s}^{-1} \end{aligned}$$

Collision time is given as:

$$\begin{aligned} T &= d / v_{rms} \\ &= 2 \times 10^{-10} / 508.26 = 3.93 \times 10^{-13} \text{ s} \end{aligned}$$

Time taken between successive collisions:

$$\begin{aligned} T' &= l / v_{rms} \\ &= 1.11 \times 10^{-7} / 508.26 = 2.18 \times 10^{-10} \text{ s} \\ \therefore T' / T &= 2.18 \times 10^{-10} / (3.93 \times 10^{-13}) = 500 \end{aligned}$$

Hence, the time taken between successive collisions is 500 times the time taken for a collision.

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Additional Exercises

13.11. A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Answer

Length of the narrow bore, $L = 1 \text{ m} = 100 \text{ cm}$

Length of the mercury thread, $l = 76 \text{ cm}$

Length of the air column between mercury and the closed end, $l_a = 15$ cm

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: $100 - (76 + 15) = 9$ cm

Hence, the total length of the air column = $15 + 9 = 24$ cm

Let h cm of mercury flow out as a result of atmospheric pressure.

\therefore Length of the air column in the bore = $24 + h$ cm

And, length of the mercury column = $76 - h$ cm

Initial pressure, $P_1 = 76$ cm of mercury

Initial volume, $V_1 = 15$ cm³

Final pressure, $P_2 = 76 - (76 - h) = h$ cm of mercury

Final volume, $V_2 = (24 + h)$ cm³

Temperature remains constant throughout the process.

$\therefore P_1V_1 = P_2V_2$

$76 \times 15 = h(24 + h)$

$h^2 + 24h - 1140 = 0$

$$\therefore h = \frac{-24 \pm (24)^2 + 4 \times 1 \times 1140}{2 \times 1}$$

= 23.8 cm or -47.8 cm

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be $24 + 23.8 = 47.8$ cm.

13.12. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 cm³ s⁻¹. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 cm³ s⁻¹. Identify the gas.

[Hint: Use Graham's law of diffusion: $R_1/R_2 = (M_2/M_1)^{1/2}$, where R_1, R_2 are diffusion rates of gases 1 and 2, and M_1 and M_2 their respective molecular masses. The law is a simple consequence of kinetic theory.]

Answer

Rate of diffusion of hydrogen, $R_1 = 28.7$ cm³ s⁻¹

Rate of diffusion of another gas, $R_2 = 7.2$ cm³ s⁻¹

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

where,

M_1 is the molecular mass of hydrogen = 2.020 g

M_2 is the molecular mass of the unknown gas

$$\begin{aligned}\therefore M_2 &= M_1 \left(\frac{R_1}{R_2}\right)^2 \\ &= 2.02 \left(\frac{28.7}{7.2}\right)^2 = 32.09 \text{ g}\end{aligned}$$

32 g is the molecular mass of oxygen. Hence, the unknown gas is oxygen.

13.13. A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres

$$n_2 = n_1 \exp [-mg (h_2 - h_1) / k_B T]$$

Where n_2 , n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column:

$$n_2 = n_1 \exp [-mg N_A (\rho - \rho') (h_2 - h_1) / (\rho R T)]$$

Where ρ is the density of the suspended particle, and ρ' that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant.] [Hint: Use Archimedes principle to find the apparent weight of the suspended particle.]

Answer

According to the law of atmospheres, we have:

$$n_2 = n_1 \exp [-mg (h_2 - h_1) / k_B T] \dots \text{(i)}$$

where,

n_1 is the number density at height h_1 , and n_2 is the number density at height h_2

mg is the weight of the particle suspended in the gas column

Density of the medium = ρ'

Density of the suspended particle = ρ

Mass of one suspended particle = m'

Mass of the medium displaced = m

Volume of a suspended particle = V

According to Archimedes' principle for a particle suspended in a liquid column, the effective weight of the suspended particle is given as:

Weight of the medium displaced – Weight of the suspended particle

$$= mg - m'g$$

$$= mg - V \rho' g = mg - (m/\rho)\rho'g$$

$$= mg(1 - (\rho'/\rho)) \dots(\text{ii})$$

Gas constant, $R = k_B N$

$$k_B = R / N \dots(\text{iii})$$

Substituting equation (ii) in place of mg in equation (i) and then using equation (iii), we get:

$$n_2 = n_1 \exp [-mg (h_2 - h_1) / k_B T]$$

$$= n_1 \exp [-mg (1 - (\rho'/\rho)) (h_2 - h_1) (N/RT)]$$

$$= n_1 \exp [-mg (\rho - \rho')(h_2 - h_1) (N/RT\rho)]$$

13.14. Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms:

Substance	Atomic Mass (u)	Density (10^3 Kg m^{-3})
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Hint: Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

Answer

If r is the radius of the atom, then volume of each atom = $4/3 \pi r^3$

Volume of all atoms in one mole of substance = $4/3 \pi r^3 \times N = M/\rho$

$$\therefore r = [3M / 4\pi\rho N]^{1/3}$$

For Carbon,

$$M = 12.01 \times 10^{-3} \text{ Kg}$$

$$\rho = 2.22 \times 10^3 \text{ Kg m}^{-3}$$

$$r = \frac{3 \times 12.01 \times 10^{-3}}{4 \times \frac{22}{7} \times (2.22 \times 10^3) \times (6.023 \times 10^{23})}$$

$$= 1.29 \times 10^{-10} \text{ m}$$

$$= 1.29 \text{ Å}$$

Similarly,

for gold, $r = 1.59 \text{ Å}$

for liquid nitrogen, $r = 1.77 \text{ Å}$



for lithium, $r = 1.73 \text{ \AA}$

for liquid fluorine, $r = 1.88 \text{ \AA}$

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