## JAIPUR EDUCATION PLUS

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## ANSWERS

## SECTION - A

## 1. Solution:

Since the given graph intersects the $x$-axis at 3 points, the polynomial has 3 zeros.
2. Solution:

We have

```
LHS = tan1 }\mp@subsup{}{}{\circ}\operatorname{tan}\mp@subsup{2}{}{\circ}\operatorname{tan}\mp@subsup{3}{}{\circ}\ldots.....\operatorname{tan}8\mp@subsup{9}{}{\circ
    = tan1 }\mp@subsup{1}{}{\circ}\operatorname{tan}\mp@subsup{2}{}{\circ}\operatorname{tan}\mp@subsup{3}{}{\circ}\ldots..\operatorname{tan}4\mp@subsup{4}{}{\circ}\operatorname{tan}4\mp@subsup{5}{}{\circ}\operatorname{tan}4\mp@subsup{6}{}{\circ}\ldots..\operatorname{tan}8\mp@subsup{8}{}{\circ}\operatorname{tan}8\mp@subsup{9}{}{\circ
    = (tan1'.tan89}\mp@subsup{}{}{\circ})(\operatorname{tan}\mp@subsup{2}{}{\circ}\cdot\operatorname{tan}8\mp@subsup{8}{}{\circ})\ldots(\operatorname{tan}4\mp@subsup{4}{}{\circ}\operatorname{tan}4\mp@subsup{6}{}{\circ})\operatorname{tan}4\mp@subsup{5}{}{\circ
    = {tan1\circ}\cdot\operatorname{tan}(9\mp@subsup{0}{}{\circ}-\mp@subsup{1}{}{\circ})}{(\operatorname{tan}\mp@subsup{2}{}{\circ}\cdot\operatorname{tan}(9\mp@subsup{0}{}{\circ}-\mp@subsup{2}{}{\circ})}\ldots{(..{\operatorname{tan}4\mp@subsup{4}{}{\circ}\operatorname{tan}(9\mp@subsup{0}{}{\circ}-4\mp@subsup{4}{}{\circ})}\operatorname{tan}4\mp@subsup{5}{}{\circ
    ={tan1 }\mp@subsup{1}{}{\circ}\cdot\operatorname{cot}\mp@subsup{1}{}{\circ})(\operatorname{tan}\mp@subsup{2}{}{\circ}\cdot\operatorname{cot}\mp@subsup{2}{}{\circ})\ldots(\operatorname{tan}4\mp@subsup{4}{}{\circ}\operatorname{cot}4\mp@subsup{4}{}{\circ})\operatorname{tan}4\mp@subsup{5}{}{\circ}\quad[\because\operatorname{tan}(9\mp@subsup{0}{}{\circ}-0)=\operatorname{cot}0
    = 1 = RHS
[\because\operatorname{tan}0\operatorname{cot}0=1 and tan45 利 1]
```


## 3. Solution:

Let $x=0 . \overline{6}$.
Then, $x=0.666$
$\therefore \quad 10 x=6.666$
On subtracting (i) from (ii), we get

$$
9 x=6 \Rightarrow \quad x=\frac{6}{9}=\frac{2}{3}
$$

Thus, $0 . \overline{6}=\frac{2}{3}$

## 4. Solution:

$$
\begin{aligned}
\sec 67^{\circ}+\operatorname{cosec} 58^{\circ} & =\sec \left(90^{\circ}-23^{\circ}\right)+\operatorname{cosec}\left(90^{\circ}-32^{\circ}\right) \\
& =\operatorname{cosec} 23^{\circ}+\sec 32^{\circ}
\end{aligned}
$$

5. Solution:

Since AE is the bisector of the exterior $\angle \mathrm{CAD}$.

$$
\therefore \quad \frac{\mathrm{BE}}{\mathrm{CE}}=\frac{\mathrm{AB}}{\mathrm{AC}} \quad \Rightarrow \quad \frac{12+x}{x}=\frac{10}{6} \quad \Rightarrow \quad x=18
$$

## SECTION - B

## 6. Solution:

We have,

$$
\begin{aligned}
p(x)= & 2 x^{2}-3 x+4 \\
\Rightarrow \quad p(3) & =2 \times 3^{2}-3 \times 3+4 \\
& =2 \times 9-9+4 \\
& =18-9+4=22-9=13 \\
p(-1) & =2 \times(-1)^{2}-3 \times(-1)+4 \\
& =2+3+4 \\
& =9
\end{aligned}
$$

## 7. Solution:

Let the larger number be $x$ and the smaller number be $y$.
Then, $x+y=1000$
And, $\quad x^{2}-y^{2}=256000$
On dividing (ii) by (i), we get

$$
\begin{align*}
& \frac{x^{2}-y^{2}}{x+y}=\frac{256000}{1000} \\
\Rightarrow \quad & x-y=256 \tag{iii}
\end{align*}
$$

Adding (i) and (iii), we get

$$
\begin{aligned}
& 2 x=1256 \\
\Rightarrow \quad & x=628
\end{aligned}
$$

Substituting $x=628$ in (i), we get $y=372$
Thus, the required numbers are 628 and 372 .

## 8. Solution:

$$
\begin{aligned}
& \mathrm{A}=60^{\circ} \text { and } \mathrm{B}=30^{\circ} \\
\Rightarrow \quad & \mathrm{A}-\mathrm{B}=60^{\circ}-30^{\circ}=30^{\circ} \\
\therefore \quad & \sin (\mathrm{A}-\mathrm{B})=\sin 30^{\circ}=\frac{1}{2} \\
\sin A \cos \mathrm{~B}-\cos \mathrm{A} \sin \mathrm{~B} & =\sin 60^{\circ} \cos 30^{\circ}-\cos 60^{\circ} \sin 30^{\circ} \\
& =\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}-\frac{1}{2} \times \frac{1}{2}
\end{aligned}
$$

$$
=\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}
$$

$\therefore \quad \sin (A-B)=\sin A \cos B-\cos A \sin B$

## 9. Solution:

Let the shortest side be $x$ metres in length. Then,
Hypotenuse $=(2 x+6) \mathrm{m}$ and third side $=(2 x+4) \mathrm{m}$
By Pythagoras theorem, we have

$$
\begin{array}{ll} 
& (2 x+6)^{2}=x^{2}+(2 x+4)^{2} \\
\Rightarrow & 4 x^{2}+24 x+36=x^{2}+4 x^{2}+16 x+16 \\
\Rightarrow & x^{2}-8 x-20=0 \\
\Rightarrow & (x-10)(x+2)=0 \\
\Rightarrow & x=10 \text { or } x=-2 \\
\Rightarrow & x=10
\end{array}
$$

Thus, the sides of the triangles are $10 \mathrm{~m}, 26 \mathrm{~m}$ and 24 m .

## 10. Solution:

We observe that the class 1500-2000 has the maximum frequency 40. So, it is the modal class such that

$$
\begin{aligned}
l=1500, h=500, f=40, f_{1}= & 24 \text { and } f_{2}=33 \\
\therefore \quad \text { Mode }=l+\frac{f-f_{1}}{2 f-f_{1}-f_{2}} \times h= & 1500+\frac{40-24}{80-24-33} \times 500 \\
& =1500+\frac{16}{23} \times 500=1847.826
\end{aligned}
$$

## SECTION - C

## 11. Solution:

Let the required numbers be $x$ and $y$.
Then, $x+y=16$
And, $\frac{1}{x}+\frac{1}{y}=\frac{1}{3} \quad \Rightarrow \quad \frac{x+y}{x y}=\frac{1}{3} \quad \Rightarrow \quad \frac{16}{x y}=\frac{1}{3} \quad \Rightarrow \quad x y=48$
We can write

$$
x-y=\sqrt{(x+y)^{2}-4 x y}
$$

$$
\begin{align*}
&=\sqrt{(16)^{2}-4 \times 48} \\
&=\sqrt{256-192}=\sqrt{64}= \pm 8 \\
& \therefore \quad x+y= 16  \tag{i}\\
& \text { Or, } \quad x+y= 8  \tag{ii}\\
& x+y= 16  \tag{iii}\\
& x-8 \tag{iv}
\end{align*}
$$

On solving (i) and (ii), we get $x=12$ and $y=4$
On solving (iii) and (iv), we get $x=4$ and $y=12$
Thus, the required numbers are 12 and 4 .

## 12. Solution:

Let $A B C$ be an equilateral triangle inscribed in a circle of radius 6 cm . Let 0 be the centre of the circle. Then,

$$
\mathrm{OA}=\mathrm{OB}=\mathrm{OC}=6 \mathrm{~cm}
$$

Let OD be perpendicular from $O$ on side $B C$. Then, $D$ is the mid-point of $B C$ and $O B$ and $O C$ are bisectors of $\angle \mathrm{B}$ and $\angle \mathrm{C}$ respectively.

$$
\therefore \quad \angle \mathrm{OBD}=30^{\circ}
$$

In $\triangle \mathrm{OBD}$, right angled at D , we have

$$
\begin{array}{ll} 
& \angle O B D=30^{\circ} \text { and } O B=6 \mathrm{~cm} \\
\therefore & \cos \angle \mathrm{OBD}=\frac{\mathrm{BD}}{\mathrm{OB}} \\
\Rightarrow & \cos 30^{\circ}=\frac{\mathrm{BD}}{6} \\
\Rightarrow & \mathrm{BD}=6 \cos 30^{\circ} \\
\Rightarrow & \mathrm{BD}=6 \times \frac{\sqrt{3}}{2}=3 \sqrt{3} \\
\Rightarrow & \quad \mathrm{BC}=2 \mathrm{BD}=2 \times 3 \sqrt{3}=6 \sqrt{3}
\end{array}
$$

Thus, the side of the equilateral triangle is $6 \sqrt{3}$.

## 13. Solution:

We have,

$$
\sin \theta=\frac{a}{\sqrt{a^{2}+b^{2}}}
$$

$$
\begin{aligned}
\therefore \quad \cos \theta & =\sqrt{1-\sin ^{2} \theta} \\
& =\sqrt{1-\frac{a^{2}}{a^{2}+b^{2}}} \\
& =\sqrt{\frac{b^{2}}{a^{2}+b^{2}}}=\frac{b}{\sqrt{a^{2}+b^{2}}} \\
\therefore \quad \tan \theta & =\frac{\sin \theta}{\cos \theta}=\frac{\frac{a}{\sqrt{a^{2}+b^{2}}}}{\frac{b}{\sqrt{a^{2}+b^{2}}}}=\frac{a}{b}
\end{aligned}
$$

## 14. Solution:

Let $A B C D$ be the given rectangle and let 0 be a point within it. Join $O A, O B, O C$ and $O D$.
Through 0, draw EOF || AB. Then, ABFE is a rectangle.
In right triangles OEA and OFC, we have

$$
\begin{array}{ll} 
& O A^{2}=O E^{2}+\mathrm{AE}^{2} \text { amd } O \mathrm{O}^{2}=O \mathrm{OF}^{2}+\mathrm{CF}^{2} \\
\Rightarrow \quad & \mathrm{OA}^{2}+O \mathrm{OC}^{2}=\left(\mathrm{OE}^{2}+\mathrm{AE}^{2}\right)+\left(O F^{2}+\mathrm{CF}^{2}\right) \\
\Rightarrow \quad & \mathrm{OA}^{2}+O \mathrm{OC}^{2}=O \mathrm{OE}^{2}+O \mathrm{OF}^{2}+\mathrm{AE}^{2}+\mathrm{CF}^{2} \tag{i}
\end{array}
$$

Now, in right triangles OFB and ODE, we have

$$
\begin{align*}
& O B^{2}=O F^{2}+\mathrm{FB}^{2} \text { and } \mathrm{OD}^{2}=\mathrm{OE}^{2}+\mathrm{DE}^{2} \\
\Rightarrow \quad & O B^{2}+O \mathrm{D}^{2}=\left(\mathrm{OF}^{2}+\mathrm{FB}^{2}\right)+\left(\mathrm{OE}^{2}+\mathrm{DE}^{2}\right) \\
\Rightarrow \quad & \mathrm{OB}^{2}+O \mathrm{OD}^{2}= \\
& O E^{2}+\mathrm{OF}^{2}+\mathrm{DE}^{2}+\mathrm{BF}^{2}  \tag{ii}\\
& =O E^{2}+\mathrm{OF}^{2}+\mathrm{CF}^{2}+\mathrm{AE}^{2} \quad[\because \mathrm{DE}=\mathrm{CF} \text { and } \mathrm{AE}=\mathrm{BF}]
\end{align*}
$$

From (i) and (ii), we get

$$
\mathrm{OA}^{2}+\mathrm{OC}^{2}=\mathrm{OB}^{2}+\mathrm{OD}^{2}
$$

## 15. Solution:

Let the larger number be $x$ and smaller one be $y$. We know that

$$
\begin{equation*}
\text { Dividend }=(\text { Divisor } \times \text { Quotient })+\text { Remainder } \tag{i}
\end{equation*}
$$

When $3 x$ is divided by $y$, we get 4 as quotient and 3 as remainder. Therefore, by using (i), we get

$$
\begin{equation*}
3 x=4 y+3 \quad \Rightarrow \quad 3 x-4 y-3=0 \tag{ii}
\end{equation*}
$$

When $7 y$ is divided by $x$, we get 5 as quotient and 1 as remainder. Therefore, by using (i), we get

$$
\begin{equation*}
7 y=5 x+1 \quad \Rightarrow \quad 5 x-7 y+1=0 \tag{iii}
\end{equation*}
$$

Solving equations (ii) and (iii), by cross-multiplication, we get

$$
\begin{aligned}
& \frac{x}{-4-21}=\frac{-y}{3+15}=\frac{1}{-21+20} \\
\Rightarrow \quad & x=25 \text { and } y=18
\end{aligned}
$$

Thus, the required numbers are 25 and 18.

## 16. Solution:

Let $y=f(x)$ or $y=-4 x^{2}+4 x-1$
The following table gives the values of y for various values of x .

| $\boldsymbol{x}$ | $\mathbf{- 2}$ | $-\frac{3}{2}$ | $\mathbf{- 1}$ | $-\frac{1}{2}$ | $\mathbf{0}$ | $\frac{1}{2}$ | $\mathbf{1}$ | $\frac{3}{2}$ | $\mathbf{2}$ | $\frac{5}{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}=\mathbf{- 4 \boldsymbol { x } ^ { 2 }} \mathbf{+ 4 x - 1}$ | $\mathbf{- 2 5}$ | $\mathbf{- 1 6}$ | $\mathbf{- 9}$ | $\mathbf{- 4}$ | $\mathbf{- 1}$ | $\mathbf{0}$ | $\mathbf{- 1}$ | $\mathbf{- 4}$ | $\mathbf{- 9}$ | $\mathbf{- 1 6}$ | $\mathbf{- 2 5}$ |

Thus, the following points lie on the graph of $y=-4 x^{2}+4 x-1$ :

$$
(-2,-25),\left(-\frac{3}{2},-16\right),(-1,-9),\left(-\frac{1}{2},-4\right),(0,-1),\left(\frac{1}{2}, 0\right),(1,-1),\left(\frac{3}{2},-4\right),(2,-9),\left(\frac{5}{2},-16\right)
$$

and $(3,-25)$.
Plot these points on a graph paper and draw a free hand smooth curve passing through these points.


The shape of the curve is shown in the figure. It is a parabola opening downward having its vertex at $\left(\frac{1}{2}, 0\right)$.

## 17. Solution:

Let $n, n+1$ and $n+2$ be three consecutive positive integers.
We know that $n$ is of the form $3 q, 3 q+1$ or $3 q+2$.
So, we have the following cases:
Case I: When $n=3 q$
In this case, $n$ is divisible by 3 but $n+1$ and $n+2$ are not divisible by 3 .
Case II: When $n=3 q+1$
In this case, $n+2=3 q+1+2=3(q+1)$,
which is divisible by 3 but $n$ and $n+1$ are not divisible by 3 .
Case III: When $n=3 q+2$
In this case, $n+1=3 q+1+2=3(q+1)$,
which is divisible by 3 but $n$ and $n+2$ are not divisible by 3 .
Thus, one of $n, n+1$ and $n+2$ is divisible by 3 .
18. Solution:

We have,

$$
\begin{aligned}
& \text { LHS }=\left(m^{2}+n^{2}\right) \cos ^{2} \beta \\
&=\left(\frac{\cos ^{2} \alpha}{\cos ^{2} \beta}+\frac{\cos ^{2} \alpha}{\sin ^{2} \beta}\right) \cos ^{2} \beta \\
&=\left(\frac{\cos ^{2} \alpha \sin ^{2} \beta+\cos ^{2} \alpha \cos ^{2} \beta}{\cos ^{2} \beta \sin ^{2} \beta}\right) \cos ^{2} \beta \\
&=\cos ^{2} \alpha\left(\frac{\sin ^{2} \beta+\cos ^{2} \beta}{\cos ^{2} \beta \sin ^{2} \beta}\right) \cos ^{2} \beta \\
&=\cos ^{2} \alpha\left(\frac{1}{\cos \beta} \text { and } n=\frac{\cos \alpha}{\sin \beta}\right] \\
&=\frac{\cos ^{2} \beta \sin ^{2} \beta}{\sin ^{2} \beta}=\left(\frac{\cos \alpha}{\sin \beta}\right)^{2}=n^{2}=\text { RHS }
\end{aligned}
$$

## 19. Solution:

We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

| Class intervals | Frequency (f) | Cumulative frequency (cf) |
| :---: | :---: | :---: |
| $20-30$ | 4 | 4 |
| $30-40$ | 12 | 16 |
| $40-50$ | 14 | 30 |
| $50-60$ | 16 | 46 |
| $60-70$ | 20 | 66 |
| $70-80$ | 16 | 82 |
| $80-90$ | 10 | 92 |
| $90-100$ | 8 | 100 |
| $\mathrm{~N}=\Sigma f_{i}=100$ |  |  |

Here, $\mathrm{N}=\Sigma f_{i}=100 \Rightarrow \frac{N}{2}=50$
We observe that the cumulative frequency just greater than $\frac{N}{2}=50$ is 66 and the corresponding class is 60-70.

So, 60-70 is the median class.

$$
\therefore \quad l=60, f=20, \mathrm{~F}=46 \text { and } h=10
$$

Now, median $=1+\frac{\frac{N}{2}-F}{f} \times h$

$$
=60+\frac{50-46}{20} \times 10=62
$$

## 20. Solution:

Given: Three parallel lines $\mathrm{l}, \mathrm{m}$ and n which are cut by the transversals AB and CD in $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and E, F, G, respectively.

To prove: $\frac{P Q}{Q R}=\frac{E F}{F G}$

Construction: Draw PL || CD meeting the lines $m$ and $n$ in M and L , respectively.
Proof: Since PE || MF and PM || EF,
$\therefore \quad$ PMFE is a parallelogram.
$\Rightarrow \quad \mathrm{PM}=\mathrm{EF}$


Also, MF || LG and ML || FG,
$\therefore \quad$ MLFG is a parallelogram.
$\Rightarrow \quad \mathrm{ML}=\mathrm{FG}$
Now, in $\triangle$ PRL, we have
QM || RL
$\Rightarrow \quad \frac{\mathrm{PQ}}{\mathrm{QR}}=\frac{\mathrm{PM}}{\mathrm{ML}}$
[By Thale's Theorem]
$\Rightarrow \quad \frac{\mathrm{PQ}}{\mathrm{QR}}=\frac{\mathrm{EF}}{\mathrm{FG}}$

## SECTION - D

## 21. Solution:

Let $p$ be a prime number and if possible, let $\sqrt{p}$ be rational.
Let its simplest form be $\sqrt{p}=\frac{m}{n}$, where m and n are integers having no common factor other than 1 , and $n \neq 0$.

Then $\sqrt{p}=\frac{m}{n}$
$\Rightarrow \quad p=\frac{m^{2}}{n^{2}}$
[On squaring both sides]
$\Rightarrow \quad p n^{2}=m^{2}$
$\Rightarrow \quad p$ divides $m^{2}$
[ $\because p$ divides $p n^{2}$ ]
$\Rightarrow \quad p$ divides $m$
$\left[\because p\right.$ is prime and $p$ divides $m^{2} \Rightarrow p$ divides $m$ ]
Let $m=p q$ for some integer $q$.
Putting $m=p q$ in (i), we get

$$
\begin{array}{ll} 
& p n^{2}=p^{2} q^{2} \\
\Rightarrow & n^{2}=p q^{2} \\
\Rightarrow \quad & p \text { divides } n^{2} \\
\Rightarrow \quad & p \text { divides } n
\end{array}
$$

[ $\because p$ divides $p q^{2}$ ]
$\left[\because p\right.$ is prime and $p$ divides $n^{2} \Rightarrow p$ divides $\left.n\right]$

Thus, $p$ is a common factor of $m$ and $n$.
But this contradicts the fact that $m$ and $n$ have no common factor other than 1 .

The contradiction arises by assuming that $\sqrt{p}$ is rational.
Thus, $\sqrt{p}$ is irrational.

## 22. Solution:

Let the assumed mean be $\mathrm{A}=50$ and $h=20$.
Calculation of mean

| Class | Frequency <br> $f_{i}$ | Mid-values | $u_{i}=\frac{x_{i}-A}{h}$ | $f_{i} u_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| $0-20$ | 17 | 10 | -2 | -34 |
| $20-40$ | $f_{1}$ | 30 | -1 | $-f_{1}$ |
| $40-60$ | 32 | 50 | 0 | 0 |
| $60-80$ | $f_{2}$ | 70 | 1 | $f_{2}$ |
| $80-100$ | 19 | 90 | 2 | 38 |
| $\mathrm{~N}=\sum f_{i}=68+f_{1}+f_{2}$ |  |  |  |  |

We have,

$$
\begin{align*}
& \mathrm{N}=\sum f_{i}=120 \\
\Rightarrow \quad & 68+f_{1}+f_{2}=120 \\
\Rightarrow \quad & f_{1}+f_{2}=52 \tag{i}
\end{align*}
$$

[Given]

Now,

$$
\begin{align*}
& \text { Mean }=50 \\
\Rightarrow & A+h\left\{\frac{1}{N} \sum f_{i} u_{i}\right\}=50 \\
\Rightarrow & 50+20 \times\left\{\frac{4-f_{1}+f_{2}}{120}\right\}=50 \\
\Rightarrow & 50+\frac{4-f_{1}+f_{2}}{6}=50 \\
\Rightarrow & \frac{4-f_{1}+f_{2}}{6}=0 \\
\Rightarrow & 4-f_{1}+f_{2}=0 \\
\Rightarrow & f_{1}-f_{2}=4 \tag{ii}
\end{align*}
$$

Solving equations (i) and (ii), we get

$$
f_{1}=28 \text { and } f_{2}=24
$$

## 23. Solution:

Let the speed of the boat in still water be $x \mathrm{~km} / \mathrm{hr}$ and the speed of the stream be $y \mathrm{~km} / \mathrm{hr}$. Then,

Speed upstream $=(x-y) \mathrm{km} / \mathrm{hr}$
Speed downstream $=(x+y) \mathrm{km} / \mathrm{hr}$
Now,
Time taken to cover 32 km upstream $=\frac{32}{x-y} \mathrm{hrs}$
Time taken to cover 36 km downstream $=\frac{36}{x+y} \mathrm{hrs}$
But, total time of journey is 7 hours

$$
\begin{equation*}
\therefore \quad \frac{32}{x-y}+\frac{36}{x+y}=7 \tag{i}
\end{equation*}
$$

Time taken to cover 40 km upstream $=\frac{40}{x-y}$
Time taken to cover 48 km downstream $=\frac{48}{x+y}$
In this case, total time of journey is given to be 9 hours.

$$
\begin{equation*}
\therefore \quad \frac{40}{x-y}+\frac{48}{x+y}=9 \tag{ii}
\end{equation*}
$$

Putting $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$ in equations (i) and (ii), we get

$$
\begin{array}{lll}
32 u+36 v=7 & \Rightarrow & 32 u+36 v-7=0 \\
40 u+48 v=9 & \Rightarrow & 40 u+48 v-9=0 \tag{iv}
\end{array}
$$

Solving these equations by cross-multiplication, we get

$$
\begin{aligned}
& \frac{u}{36 \times-9-48 \times-7}=\frac{-v}{32 \times-9-40 \times-7}=\frac{1}{32 \times 48-40 \times 36} \\
\Rightarrow & \frac{u}{-324+336}=\frac{-v}{-288+280}=\frac{1}{1536-1440} \\
\Rightarrow & \frac{u}{12}=\frac{v}{8}=\frac{1}{96} \\
\Rightarrow & u=\frac{12}{96}=\frac{1}{8} \text { and } v=\frac{8}{96}=\frac{1}{8}
\end{aligned}
$$

Now, $u=\frac{1}{8} \quad \Rightarrow \quad \frac{1}{x-y}=\frac{1}{8} \quad \Rightarrow \quad x-y=8$
and, $\quad v=\frac{1}{12} \quad \Rightarrow \quad \frac{1}{x+y}=\frac{1}{12} \quad \Rightarrow \quad x+y=12$
Solving equations (v) and (vi), we get $x=10$ and $y=2$
Thus, speed of the boat in still water $=10 \mathrm{~km} / \mathrm{hr}$
Speed of the stream $=2 \mathrm{~km} / \mathrm{hr}$

## 24. Solution:

Given: $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$ and AX and DY are bisector of $\angle \mathrm{A}$ and $\angle \mathrm{D}$ respectively.
To prove: $\frac{\operatorname{Area}(\triangle \mathrm{ABC})}{\operatorname{Area}(\triangle \mathrm{DEF})}=\frac{\mathrm{AX}^{2}}{\mathrm{DY}^{2}}$
Proof: Since the ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.
$\therefore \quad \frac{\operatorname{Area}(\triangle \mathrm{ABC})}{\operatorname{Area}(\triangle \mathrm{DEF})}=\frac{\mathrm{AB}^{2}}{\mathrm{DE}^{2}}$
Now, $\quad \triangle \mathrm{ABC} \sim \Delta \mathrm{DEF}$
$\Rightarrow \quad \angle \mathrm{A}=\angle \mathrm{D}$
$\Rightarrow \quad \frac{1}{2} \angle \mathrm{~A}=\frac{1}{2} \angle \mathrm{D}$
$\Rightarrow \quad \angle \mathrm{BAX}=\angle \mathrm{EDY}$


Thus, in triangles ABX and DEY, we have

$$
\angle \mathrm{BAX}=\angle \mathrm{EDY} \text { and } \angle \mathrm{B}=\angle \mathrm{E}
$$

$$
[\because \Delta \mathrm{ABC} \sim \Delta \mathrm{DEF}]
$$

So, by AA-similarity criterion, we have
$\Delta \mathrm{ABX} \sim \Delta \mathrm{DEY}$
$\Rightarrow \quad \frac{\mathrm{AB}}{\mathrm{DE}}=\frac{\mathrm{AX}}{\mathrm{DY}}$
$\Rightarrow \quad \frac{\mathrm{AB}^{2}}{\mathrm{DE}^{2}}=\frac{\mathrm{AX}^{2}}{\mathrm{DY}^{2}}$
From (i) and (ii), we get

$$
\frac{\operatorname{Area}(\triangle \mathrm{ABC})}{\operatorname{Area}(\triangle \mathrm{DEF})}=\frac{\mathrm{AX}^{2}}{\mathrm{DY}^{2}}
$$

## 25. Solution:

Let the cost of 1 pencil be Rs $x$ and that of one eraser be Rs $y$.
It is given that Rama purchased 2 pencils and 3 erasers for Rs 9 .

$$
\therefore \quad 2 x+3 y=9
$$

It is also given that Sonal purchased 4 pencils and 6 erasers for Rs 18.

$$
\therefore \quad 4 x+6 y=18
$$

Algebraic representation: The algebraic representation of the given situation is

$$
\begin{aligned}
& 2 x+3 y=9 \\
& 4 x+6 y=18
\end{aligned}
$$

Graphical representation: In order to obtain the graphical representation of the above pair of linear equations, we find two points on the line representing each equation. That is, we find two solutions of each equation. Let us find these solutions. We will try to find solutions having integral values.

We have,

$$
2 x+3 y=9
$$

Putting $x=-3$, we get

$$
-6+3 y=9 \quad \Rightarrow \quad 3 y=15 \quad \Rightarrow \quad y=5
$$

Putting $x=0$, we get

$$
0+3 y=9 \quad \Rightarrow \quad y=3
$$

Thus, two solutions of $2 x+3 y=9$ are:

| $x$ | -3 | 0 |
| :--- | :--- | :--- |
| $y$ | 5 | 3 |

We have,

$$
4 x+6 y=18
$$

Putting $x=3$, we get

$$
12+6 y=18 \Rightarrow 6 y=6 \quad \Rightarrow \quad y=1
$$

Putting $x=-6$, we get

$$
-24+6 y=18 \Rightarrow 6 y=42 \quad \Rightarrow \quad y=7
$$

Thus, two solutions of $4 x+6 y=18$ are:

| $x$ | 3 | -6 |
| :--- | :--- | :--- |
| $y$ | 1 | 7 |

Now, we plot the points $\mathrm{A}(-3,5)$ and $\mathrm{B}(0,3)$ and draw the line passing through these points to obtain the graph of the line $2 x+3 y=9$. Points $\mathrm{P}(3,1)$ and $\mathrm{Q}(-6,7)$ are plotted on the graph paper and we join them to obtain the graph of the line $4 x+6 y=18$. We find that both the lines $A B$ and $P Q$ coincide.


## 26. Solution:

Given: $A \triangle A B C$ in which $A D$ is a median.
To prove: $\mathrm{AB}^{2}+\mathrm{AC}^{2}=2 \mathrm{AD}^{2}+2\left(\frac{1}{2} \mathrm{BC}\right)^{2}$ or $\mathrm{AB}^{2}+\mathrm{AC}^{2}=2\left(\mathrm{AD}^{2}+\mathrm{BD}^{2}\right)$
Construction: Draw AE $\perp \mathrm{BC}$
Proof: Since $\angle \mathrm{AED}=90^{\circ}$, therefore, in $\triangle \mathrm{ADE}$, we have

$$
\angle \mathrm{ADE}<90^{\circ} \Rightarrow \angle \mathrm{ADB}>90^{\circ}
$$

Thus, $\triangle \mathrm{ADB}$ is an obtuse-angled triangle and $\triangle \mathrm{ADC}$ is an acute-angled triangle.
$\Delta \mathrm{ADB}$ is an obtuse-angled triangle at D and $\mathrm{AE} \perp \mathrm{BD}$ produced. Therefore, we have


$$
\begin{equation*}
\mathrm{AB}^{2}=\mathrm{AD}^{2}+\mathrm{BD}^{2}+2 \mathrm{BD} \times \mathrm{DE} \tag{i}
\end{equation*}
$$

$\triangle \mathrm{ACD}$ is an acute-angled triangle at D and $\mathrm{AE} \perp \mathrm{CD}$. Therefore, we have

$$
\begin{align*}
& \mathrm{AC}^{2}=\mathrm{AD}^{2}+\mathrm{DC}^{2}-2 \mathrm{DC} \times \mathrm{DE} \\
\Rightarrow \quad \mathrm{AC}^{2} & =\mathrm{AD}^{2}+\mathrm{BD}^{2}-2 \mathrm{BD} \times \mathrm{DE} \quad[\because \mathrm{CD}=\mathrm{BD}]
\end{align*}
$$

Adding equations (i) and (ii), we get

$$
\begin{aligned}
\mathrm{AB}^{2}+\mathrm{AC}^{2} & =2\left(\mathrm{AD}^{2}+\mathrm{BD}^{2}\right) \\
\Rightarrow \quad \mathrm{AB}^{2}+\mathrm{AC}^{2} & =2\left\{\mathrm{AD}^{2}+\left(\frac{\mathrm{BC}}{2}\right)^{2}\right\} \\
& =2 \mathrm{AD}^{2}+2\left(\frac{1}{2} \mathrm{BC}\right)^{2} \\
& =2 \mathrm{AD}^{2}+2 \mathrm{BD}^{2} \\
\Rightarrow \quad & \mathrm{AB}^{2}+\mathrm{AC}^{2}
\end{aligned}=2\left(\mathrm{AD}^{2}+\mathrm{BD}^{2}\right) \quad \$
$$

## 27. Solution:

We have,

$$
\begin{aligned}
& \mathrm{AB}=\mathrm{a} \\
\Rightarrow \quad & \mathrm{AD}+\mathrm{DB}=\mathrm{a} \\
\Rightarrow \quad & \mathrm{AD}+\mathrm{AD}=\mathrm{a} \\
\Rightarrow \quad & 2 \mathrm{AD}=\mathrm{a} \quad \Rightarrow \quad \mathrm{AD}=\frac{a}{2}
\end{aligned}
$$

Thus, $\mathrm{AD}=\mathrm{DB}=\frac{a}{2}$
By Pythagoras theorem, we have

$$
\begin{aligned}
& \mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2} \\
\Rightarrow \quad & \mathrm{~b}^{2}=\mathrm{a}^{2}+\mathrm{BC}^{2} \\
\Rightarrow \quad & \mathrm{BC}^{2}=\mathrm{b}^{2}-\mathrm{a}^{2} \\
\Rightarrow \quad & \mathrm{BC}^{2}=\sqrt{b^{2}-a^{2}}
\end{aligned}
$$

Thus, in $\triangle B C D$, we have

$$
\text { Base }=\mathrm{BC}=\sqrt{b^{2}-a^{2}} \text { and perpendicular }=\mathrm{BD}=\frac{a}{2}
$$

Applying Pythagoras theorem in $\triangle \mathrm{BCD}$, we have

$$
\mathrm{BC}^{2}+\mathrm{BD}^{2}=\mathrm{CD}^{2}
$$

$\Rightarrow \quad\left(\sqrt{b^{2}-a^{2}}\right)^{2}+\left(\frac{a}{2}\right)^{2}=\mathrm{CD}^{2}$
$\Rightarrow \quad \mathrm{CD}^{2}=b^{2}-a^{2}+\frac{a^{2}}{4}$
$\Rightarrow \quad \mathrm{CD}^{2}=\frac{4 b^{2}-4 a^{2}+a^{2}}{4}$
$\Rightarrow \quad \mathrm{CD}^{2}=\frac{4 b^{2}-3 a^{2}}{4}$
$\Rightarrow \quad \mathrm{CD}=\frac{\sqrt{4 b^{2}-3 a^{2}}}{2}$
Now, $\sin \theta=\frac{\mathrm{BD}}{\mathrm{CD}}=\frac{\frac{a}{2}}{\frac{\sqrt{4 b^{2}-3 a^{2}}}{2}}=\frac{a}{\sqrt{4 b^{2}-3 a^{2}}}$
And, $\quad \cos \theta=\frac{\mathrm{BC}}{\mathrm{CD}}=\frac{\sqrt{b^{2}-a^{2}}}{\frac{\sqrt{4 b^{2}-3 a^{2}}}{2}}=\frac{2 \sqrt{b^{2}-a^{2}}}{\sqrt{4 b^{2}-3 a^{2}}}$
Thus, $\sin ^{2} \theta+\cos ^{2} \theta=\left(\frac{a}{\sqrt{4 b^{2}-3 a^{2}}}\right)^{2}+\left(\frac{2 \sqrt{b^{2}-a^{2}}}{\sqrt{4 b^{2}-3 a^{2}}}\right)^{2}$

$$
\begin{aligned}
& =\frac{a^{2}}{4 b^{2}-3 a^{2}}+\frac{4\left(b^{2}-a^{2}\right)}{4 b^{2}-3 a^{2}} \\
& =\frac{a^{2}+4 b^{2}-4 a^{2}}{4 b^{2}-3 a^{2}} \\
& =\frac{4 b^{2}-3 a^{2}}{4 b^{2}-3 a^{2}}=1
\end{aligned}
$$

## 28. Solution:

We have,

$$
\begin{aligned}
\text { LHS } & =\frac{\cos \mathrm{A}}{1-\tan \mathrm{A}}+\frac{\sin \mathrm{A}}{1-\cot \mathrm{A}} \\
& =\frac{\cos \mathrm{A}}{1-\frac{\sin \mathrm{A}}{\cos \mathrm{~A}}}+\frac{\sin \mathrm{A}}{1-\frac{\cos \mathrm{A}}{\sin \mathrm{~A}}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}} \\
& =\frac{\cos ^{2} A}{\cos A-\sin A}+\frac{\sin ^{2} A}{\sin A-\cos A} \\
& =\frac{\cos ^{2} \mathrm{~A}}{\cos \mathrm{~A}-\sin \mathrm{A}}-\frac{\sin ^{2} \mathrm{~A}}{\cos \mathrm{~A}-\sin \mathrm{A}} \\
& =\frac{\cos ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~A}}{\cos \mathrm{~A}-\sin \mathrm{A}} \\
& =\frac{(\cos \mathrm{A}-\sin \mathrm{A})(\cos \mathrm{A}+\sin \mathrm{A})}{\cos \mathrm{A}-\sin \mathrm{A}} \\
& =\cos \mathrm{A}+\sin \mathrm{A} \\
& =\text { RHS }
\end{aligned}
$$

## 29. Solution:

Here, we are given the mid-values. So, should first find the upper and lower limits of the various classes. The difference between two consecutive values is $\mathrm{h}=125-115=10$.
$\therefore \quad$ Lower limit of a class $=$ Mid-value $-\frac{h}{2}$
Upper limit of a class $=$ Mid-value $+\frac{h}{2}$
Calculation of Median

| Mid-value | Class groups | Frequency | Cumulative frequency |
| :---: | :---: | :---: | :---: |
| 115 | $110-120$ | 6 | 6 |
| 125 | $120-130$ | 25 | 31 |
| 135 | $130-140$ | 48 | 79 |
| 145 | $140-150$ | 72 | 151 |
| 155 | $150-160$ | 116 | 267 |
| 165 | $160-170$ | 60 | 327 |
| 175 | $170-180$ | 38 | 365 |
| 185 | $180-190$ | 22 | 387 |
| 195 | $190-200$ | 3 | 390 |

We have,

$$
\mathrm{N}=390 \quad \therefore \quad \frac{\mathrm{~N}}{2}=\frac{390}{2}=195
$$

The cumulative frequency just greater than $\frac{N}{2}$, i.e., 195 is 267 and the corresponding class is 150-160. So, 150-160 is the median class.

Now,

$$
\begin{aligned}
\text { Median } & =l+\frac{\frac{N}{2}-F}{f} \times h \\
& =150+\frac{195-151}{116} \times 10=153.80
\end{aligned}
$$

30. Solution:

We have,

$$
\begin{aligned}
p(x)= & 3 x^{3}-5 x^{2}-11 x-3 \\
p(3)= & 3 \times 3^{3}-5 \times 3^{2}-11 \times 3-3 \\
& =81-45-33-3 \\
& =0 \\
p(-1)= & 3 \times(-1)^{3}-5 \times(-1)^{2}-11 \times(-1)-3 \\
& =-3-5+11-3 \\
& =0 \\
p\left(-\frac{1}{3}\right) & =3 \times\left(-\frac{1}{3}\right)^{3}-5 \times\left(-\frac{1}{3}\right)^{2}-11 \times\left(-\frac{1}{3}\right)-3 \\
& =-\frac{1}{9}-\frac{5}{9}+\frac{11}{3}-3 \\
& =0
\end{aligned}
$$

So, $3,-1$ and $-\frac{1}{3}$ are zeros of polynomial $p(x)$.
Let $\alpha=3, \beta=-1$ and $\gamma=-\frac{1}{3}$. Then,

$$
\begin{aligned}
& \alpha+\beta+\gamma=3-1-\frac{1}{3}=\frac{5}{3}=-\left(\frac{-5}{3}\right)=-\frac{\text { Coefficient of } x^{2}}{\text { Coefficient of } x^{3}} \\
& \alpha \beta+\beta \gamma+\gamma \alpha=3 \times(-1)+(-1) \times\left(-\frac{1}{3}\right)+\left(-\frac{1}{3}\right) \times 3
\end{aligned}
$$

$$
\begin{array}{r}
=-3+\frac{1}{3}-1=\frac{-11}{3}=\frac{\text { Coefficient of } x}{\text { Coefficient of } x^{3}} \\
\alpha \beta \gamma=3 \times(-1) \times\left(-\frac{1}{3}\right)=1=-\left(\frac{-3}{3}\right)=-\frac{\text { Constant term }}{\text { Coefficient of } x^{3}}
\end{array}
$$

## 31. Solution:

a. Price of petrol in December $2010=x$

Price of petrol in December $2014=y$
Price of petrol increased in 1 year $=4 \times 2=$ Rs 8
Price of petrol increased in 4 years (December 2010- December 2014) $=8 \times 4=$ Rs 32
Equation representing the price of petrol in December 2014 $=\mathrm{y}=\mathrm{x}+32$
b. Price of CNG in December $2010=a$

Price of CNG in December 2014 =b
Price of CNG increased in 1 year $=$ Rs 3
Price of CNG increased in 4 years (December 2010- December 2014) $=3 \times 4=$ Rs 12
Equation representing the price of CNG in December $2014=\mathrm{b}=\mathrm{a}+12$
c. The value depicted by using CNG over petrol is environmental protection.

